The Joy of Mathematics I

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Session 1: An introduction to numbers and something about nothing

First: something about me!

- Graduate from University of Newcastle-upon-Tyne in Mathematics from 1975
- 35 years working at the University of Liverpool in IT
- Currently ‘retired’ – writing books, training to be a Reader in the C. of E.
- Married with three grown up children

- Appeared on Countdown – sadly lost!

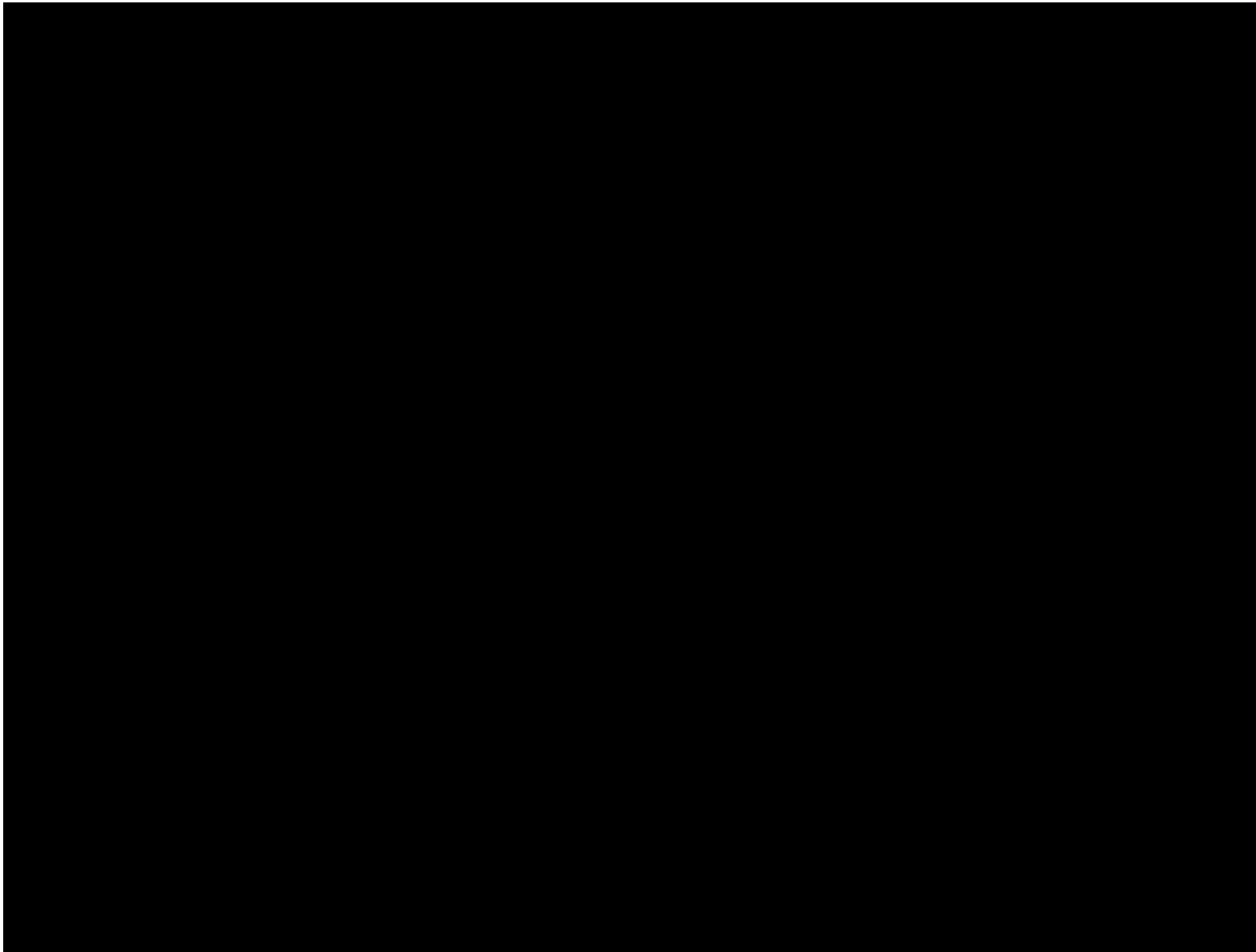
And something about you?

The Joy of Mathematics

- Recreational Mathematics i.e. maths for fun.
- Hence not a credit bearing course!
- For non-mathematicians
- Looking at ten different branches of maths
- Some exercises in and out of class
- Further reading bibliography – sources
- Online copy of all materials used at www.samdenniss.com (please register to access copyright material including these slides)

Numbers for counting: Integers

Numbers – from fish to infinity!



Let's start at the very beginning

"Maths always involves both invention *and* discovery: we invent the concepts but discover their consequences."
Stephen Strogatz, "The Joy of X" (p. 5)

4 (=2x2)

8 (=2x4)

6 (=2x3)

10 (=2x5)

3

7

5

9

5

7

5 + 7= 12

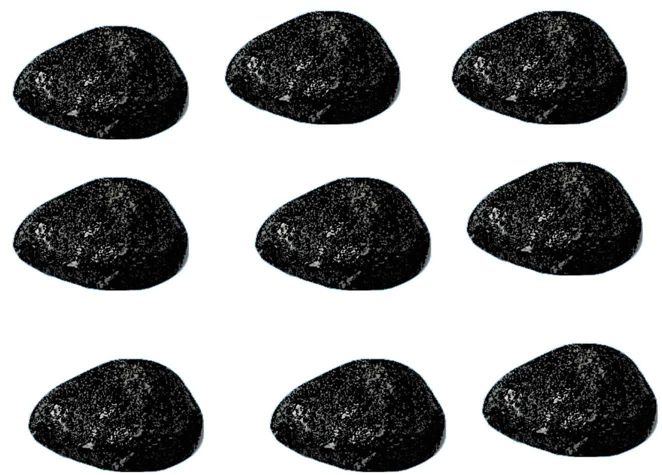
3

9

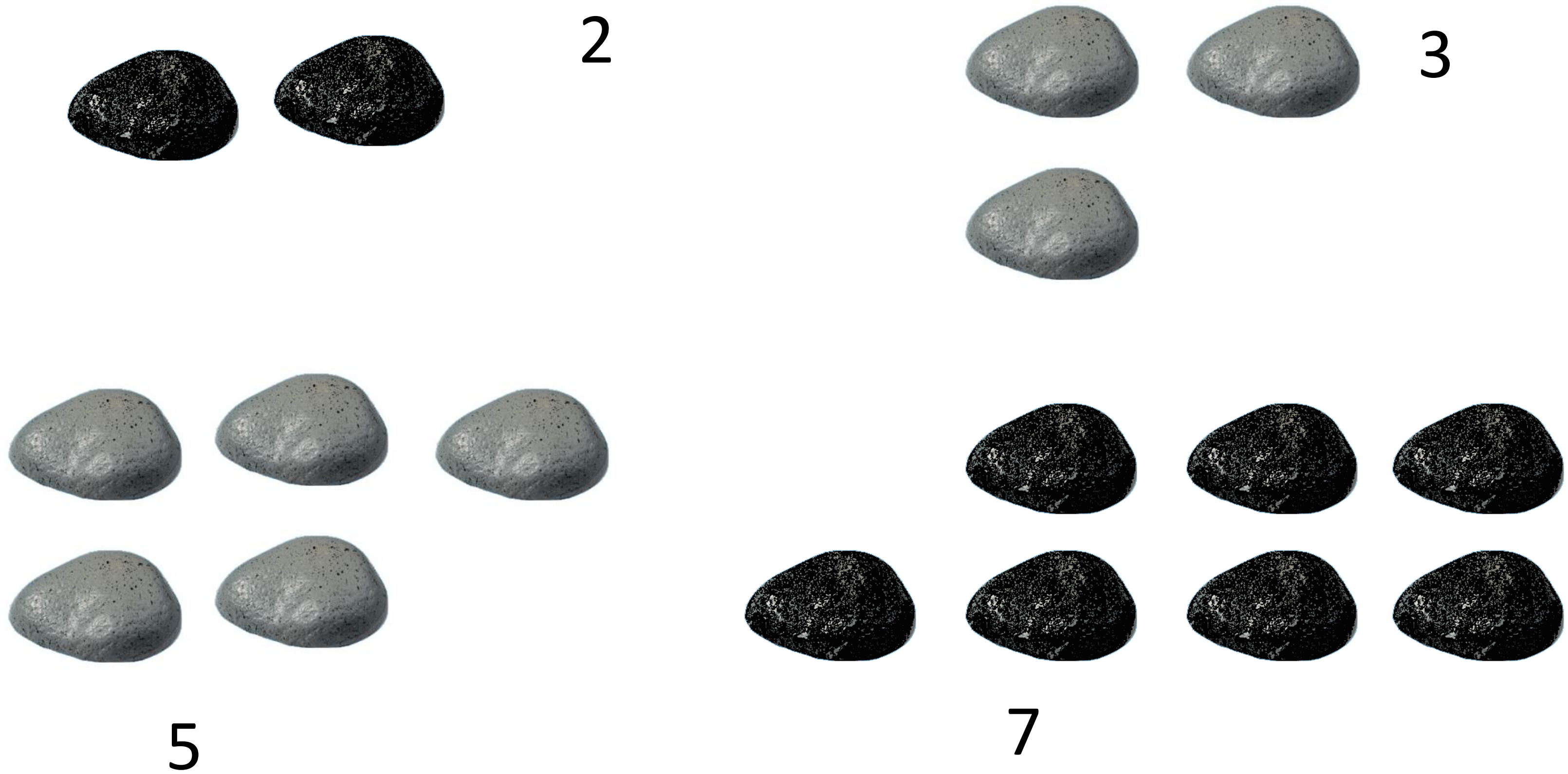
3 + 9 = 12

Add two odd numbers together to get an even number

9



9 = 3 x 3



Hopeless : can't make any kind of pattern except a single row! These are called PRIME numbers.

Prime numbers can only be divided by themselves and the number 1

The first 50 prime numbers are ....

2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97 101 103 107 109 113 127 131 137 139 149 151 157 163 167 173 179 181 191 193 197 199 211 223 227 229 233 239 241 251 257 263 269 271 277 281 283 293 307 311 313 317 331 337 347 349 353 359 367 373 379 383 389 397 401 409 419 421 431 433 439 443 449 457 461 463 467 479 487 491 499 503 509 521 523 541

http://primes.utm.edu/lists/small/1000.txt

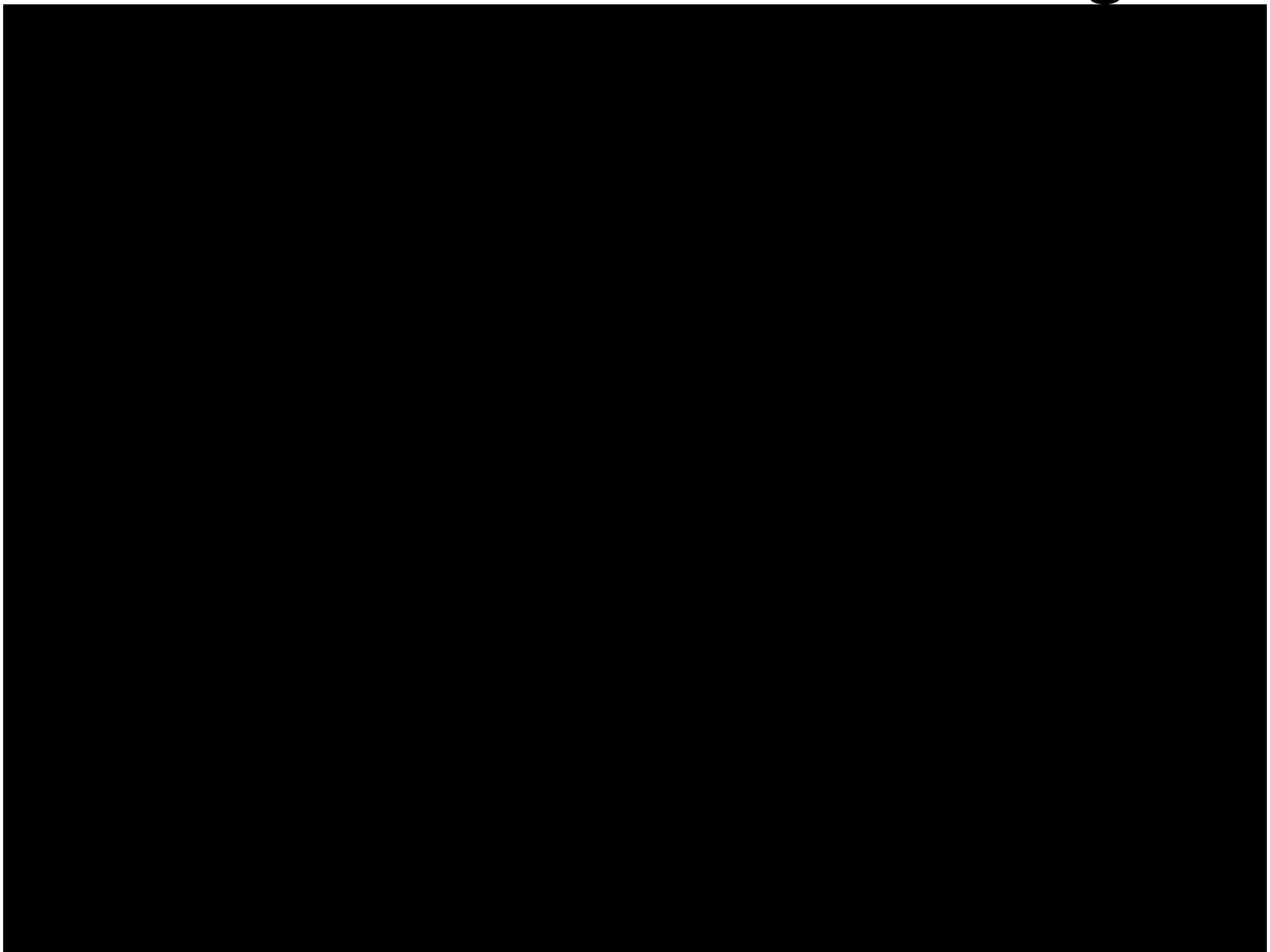
The 1000th Prime Number is 7919

You can find the first 50 million prime numbers at http://primes.utm.edu/lists/small/millions/

Find out more about primes at and some of their curious properties at:

http://primes.utm.edu/

Prime numbers in the animal kingdom



What about patterns for odd numbers?

PERFECT SQUARES

1 + 3 = 4

1+ 3 + 5 = 9

1 + 3 + 5 + 7 = 16

1 + 3 + 5 + 7 + 9 = 25

Odd numbers can be arranges in L shaped patterns

$1 + 3 = 4$

$1 + 3 + 5 = 9$

$1 + 3 + 5 + 7 = 16$

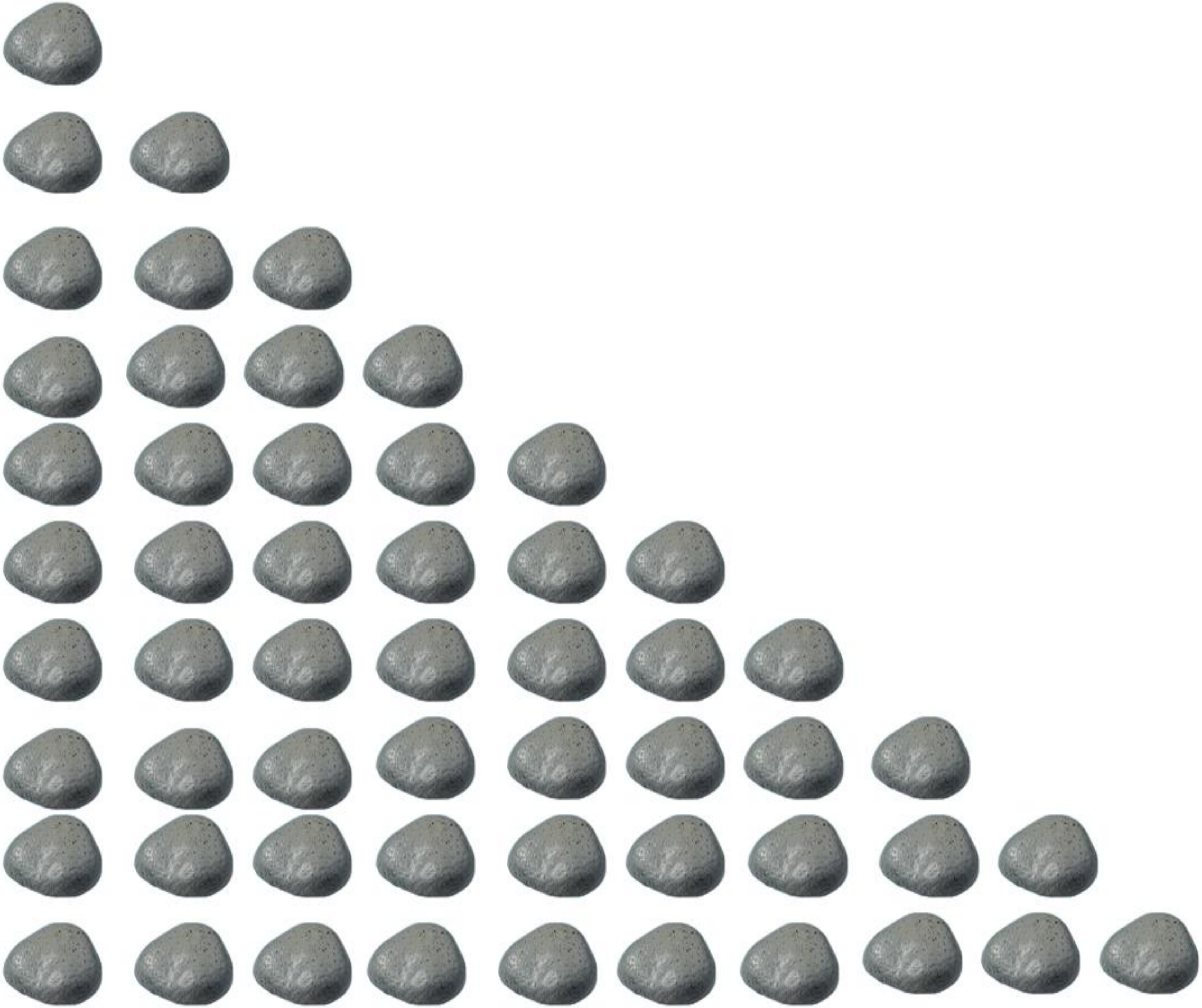
$1 + 3 + 5 + 7 + 9 = 25$

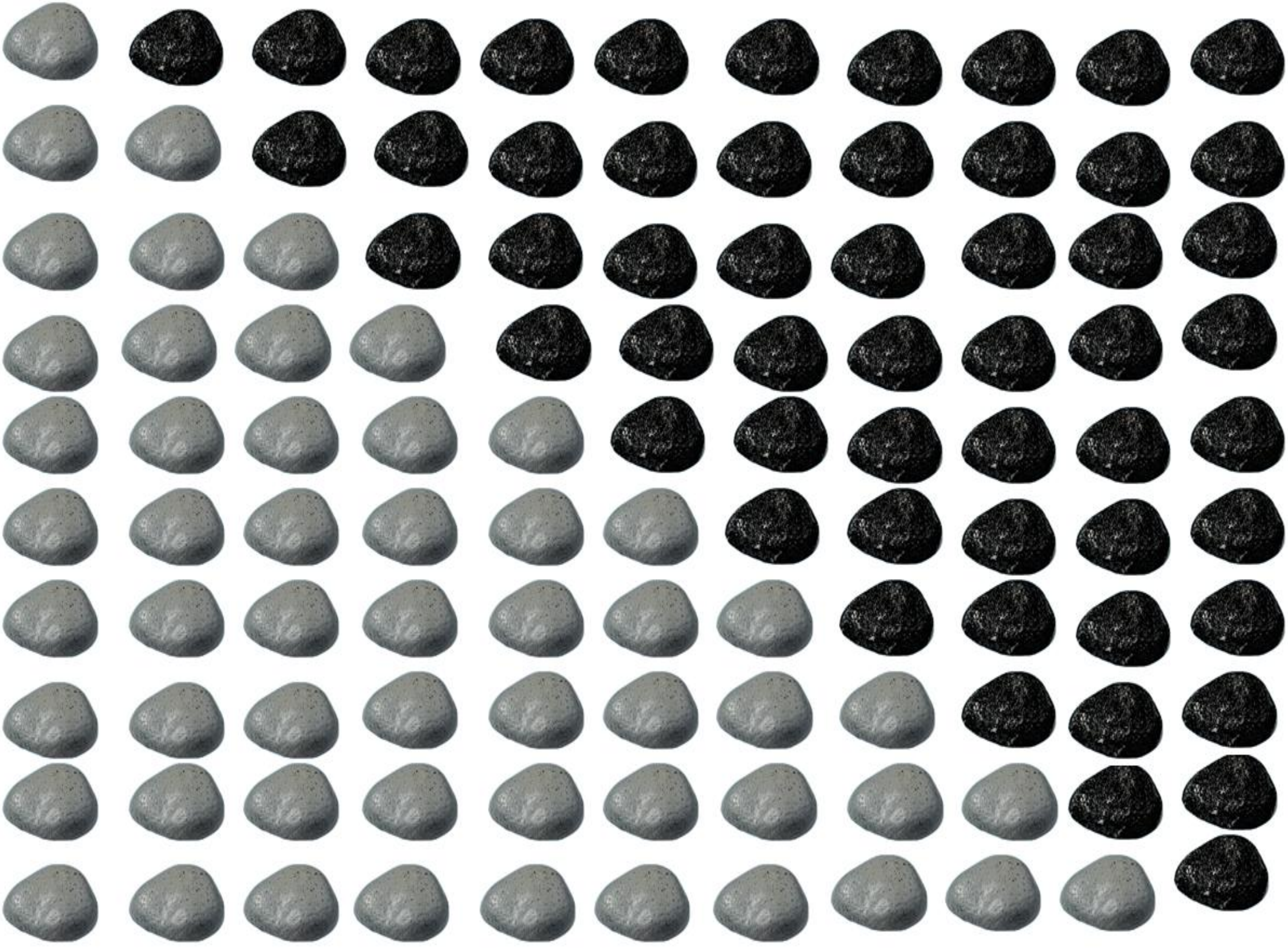
$1^2 = 1$	$5^2 = 25$	$9^2 = 81$
$2^2 = 4$	$6^2 = 36$	$10^2 = 100$
$3^2 = 9$	$7^2 = 49$	$11^2 = 121$
$4^2 = 16$	$8^2 = 64$	$12^2 = 144$

Add up the first 10 numbers, easy! Answer is 55!

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$$

Exercise: Can you give me the answer without adding them up? Hint: You need to make an appropriate pattern!





= Half of a 10 x 11 rectangle

= (10 x 11) / 2

= 55

So what is the sum of the first ‘n’ numbers? Let us put n where 10 is above!

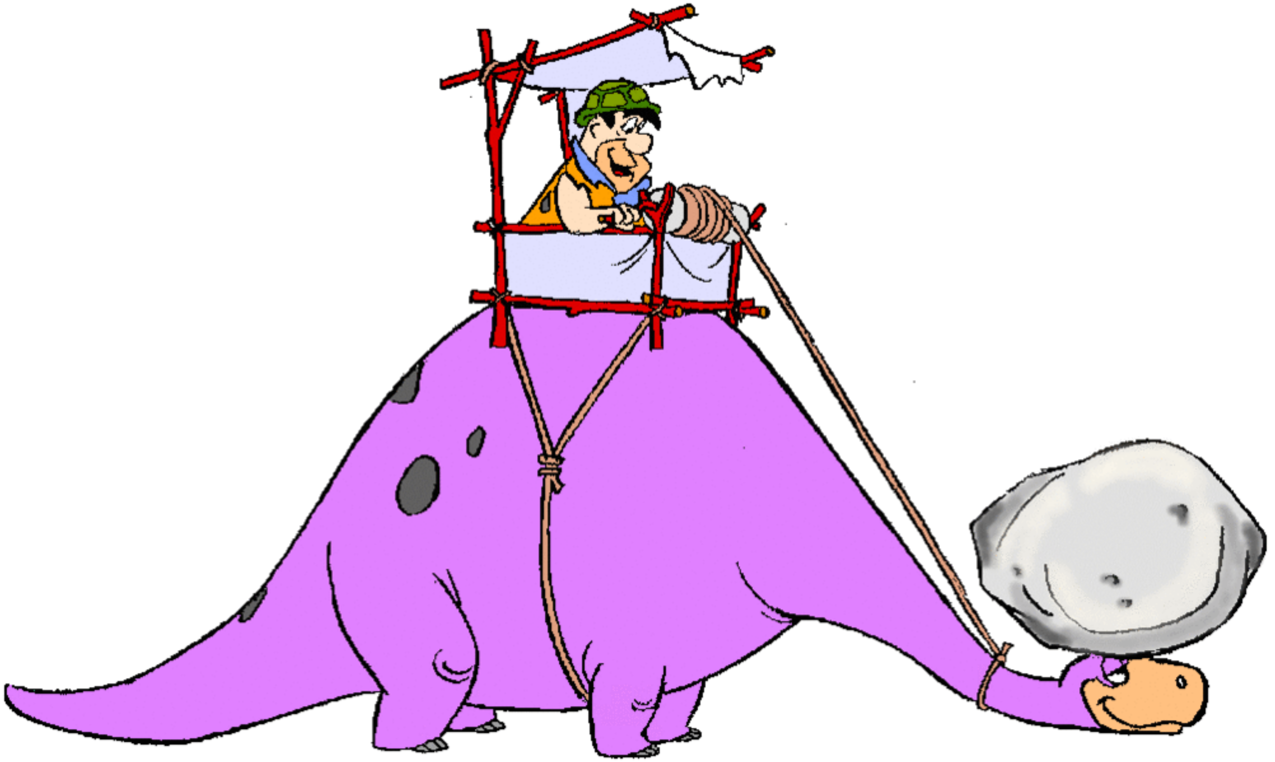
(n x (n+1)) / 2

So, where numbers are shorthand for fish, fish, fish, fish, fish

Formulae are shorthand for calculations, where there is a sum we use the Greek letter sigma and write it like this

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Calculate - counting rocks!



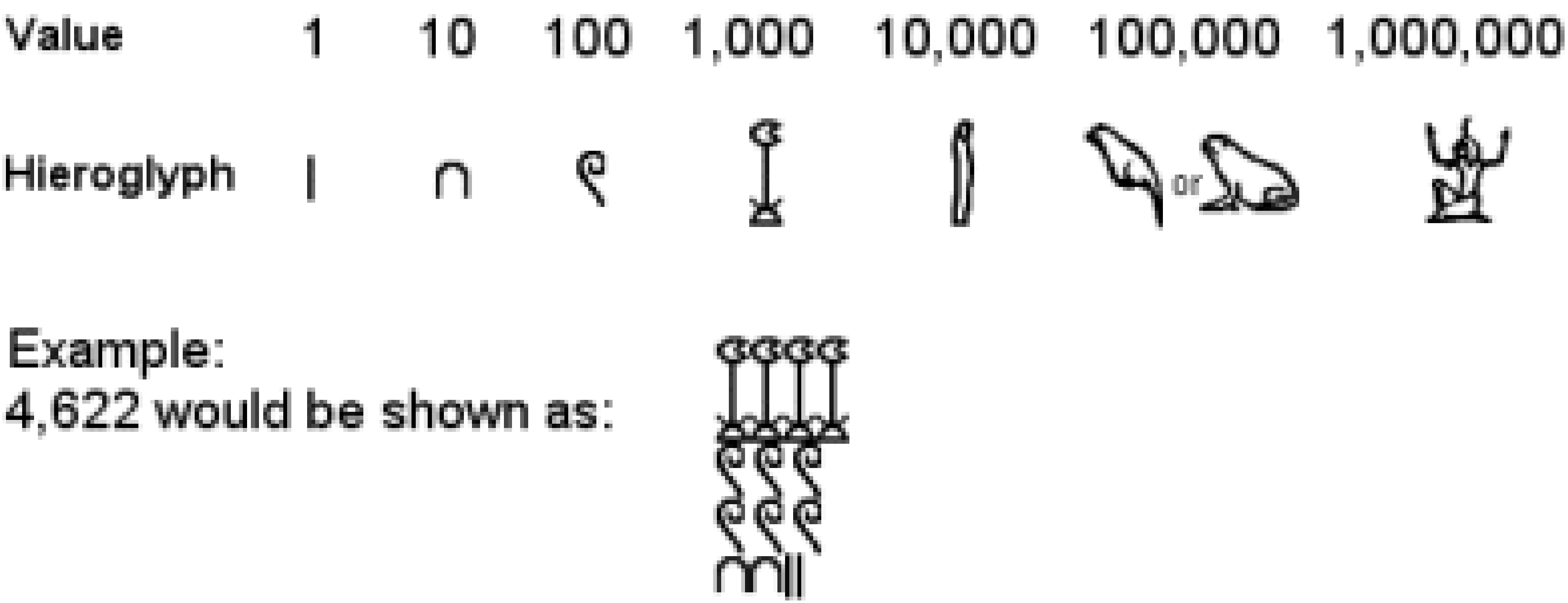
Calculus – Latin for pebble

Number systems

- Egyptians used the decimal system like us – base 10
- Babylonians used 60
- Mayans used 20
- In Greek and Hebrew letters represent numbers – it make arithmetic very hard!
- Computers use binary, base 2, sometimes numbers are written in Octal, base 8, or hexadecimal, base 16 which are ‘powers’ of 2

See number systems at http://gwydir.demon.co.uk/jo/numbers/index.htm

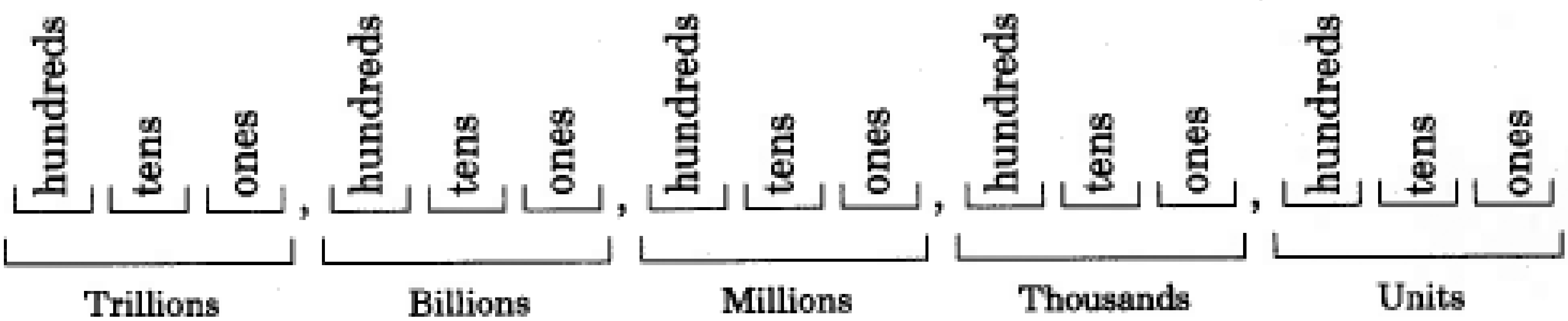
Egyptian number system



http://www.storyofmathematics.com/egyptian.html

When you have to write down a lot of fish then it starts to become a matter of plaice, sorry, place.

1,000,000



Note: A billion used to be a million millions on the British ‘long scale’
Zillion is a fictitious number!

http://cnx.org/contents/9f015c87-302a-4736-86c7-746c27594eb5@1

Number shorthand – powers!

Name	Common Notation	Math Notation	Exponent	Prefix
Quintillion	1 000 000 000 000 000 000	10^{18}	18	Exa (E)
Quadrillion	1 000 000 000 000 000	10^{15}	15	Peta (P)
Trillion	1 000 000 000 000	10^{12}	12	Tera (T)
Billion	1 000 000 000	10^{9}	9	Giga (G)
Million	1 000 000	10^{6}	6	Mega (M)
Thousand	1 000	10^{3}	3	kilo (k)
Hundred	100	10^{2}	2	hecto (h)
Ten	10	10^{1}	1	Deca (da)
One	1	10^{0}	0	
One Tenth	0.1	10^{-1}	-1	deci (d)
One Hundredth	0.01	10^{-2}	-2	centi (c)
One Thousandth	0.001	10^{-3}	-3	milli (m)
One Millionth	0.000 001	10^{-6}	-6	micro (μ)
One Billionth	0.000 000 001	10^{-9}	-9	nano (n)
One Trillionth	0.000 000 000 001	10^{-12}	-12	pico (p)
One Quadrillionth	0.000 000 000 000 001	10^{-15}	-15	femto (f)
One Quintillionth	0.000 000 000 000 000 001	10^{-18}	-18	atto (a)

Googol

1 googol = 10^{100}

1 followed by 100 0s

Other number bases

- Hexadecimal
  - 1,2,3,4,5,6,7,8,9,A,B,C,D,E,F
  - So 15 is F, 16 is 10 (or 0x10 or 10_{16})
- Octal (some Native Americans used this!)
  - 1,2,3,4,5,6,7
  - So 7 is 7, 8 is 10 (or 10o or 0o10)
- Binary
  - 0 and 1! 1, 10, 11, 100, 101, 110, 111, 1000

Easier to see binary 'pattern'

1 = 1

2= 10

3= 11

4= 100

5= 101

6= 110

7= 111

8= 1000

Binary

- Powers of 2

So 32,768 is
0b100000000000000

1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1,024
11	2,048
12	4,096
13	8,192
14	16,384
15	32,768

Write 23 in Hex, Octal and Binary

- EXERCISE.....

Write 23 in Hex, Octal and Binary

- 0x17
- 0o27
- 0b10111

the number 23

- David Beckham’s shirt number
- Movie “The Number 23”
- Wilhelm Fliess obsession with 23 (‘male cycle’) and 28 (‘female cycle’) - Biorhythms

Fliess tables e.g.
1 = (1/2 x 28) + (2x28) – (3x28)
and similar formulae for 2,3, etc up to 51 (= 23+28).

- Fleiss did not realise that any two positive integers that have no common divisor can be substituted for 23 and 28 in his basic formula! Further by finding the right values (in this case 11 and -9) you have:
  (23 x 11) + (28 x -9) = 1
  (23 x (11 x 2)) + (28 x (-9 x 2)) = 2
  (23 x (11 x 3)) + (28 x (-9 x 3)) = 3

Try it yourself with 3 and 7, what are the two numbers

3 x ? + 7 x ? = 1

Fliess (right) and Sigmund Freud in the early 1890s.

$$3 \times -2 \quad + \quad 7 \times 1 \quad = \quad 1$$

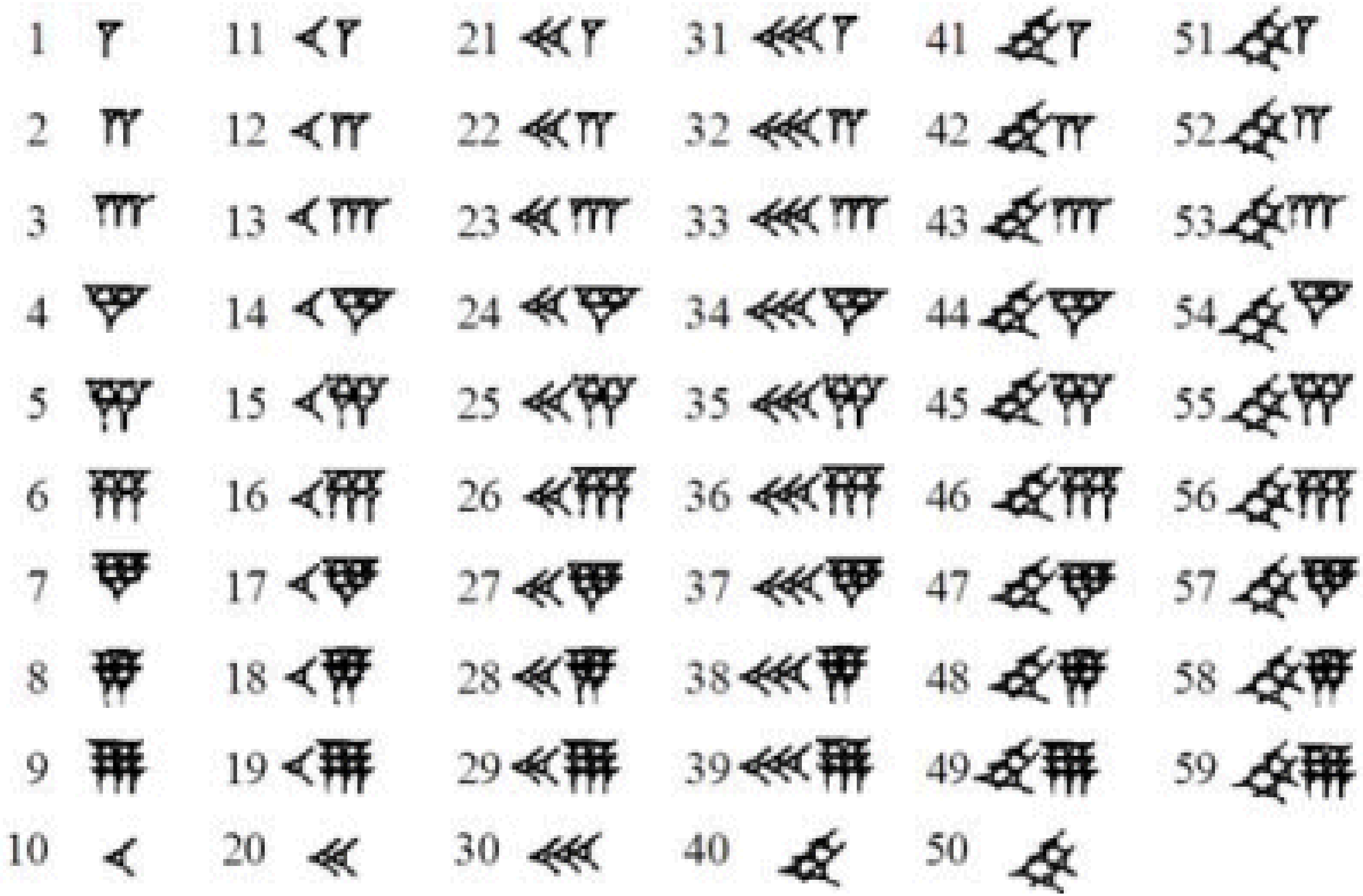
$$(3 \times (-2 \times 2)) \;+\; (7 \times (1 \times 2)) = 2$$

"*Oh emperor, my wishes are simple. I only wish for this. Give me one grain of rice for the first square of the chessboard, two grains for the next square, four for the next, eight for the next and so on for all 64 squares, with each square having double the number of grains as the square before.*"

Exponential growth!

Total number of grains = 1+2+4+8+16.......
= 18,446,744,073,709,551,615

Babylonian / Sumarian



Fish, fish, fish, fish, fish.

Take away **fish** and you are left with **fish, fish, fish fish.**

What happens when you take away all the fish, or even take away more fish than you have!

Something about nothing

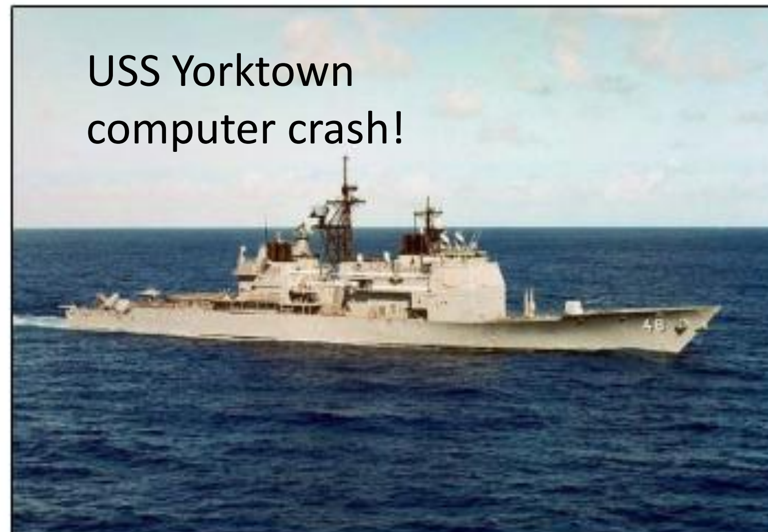
Zero – something to be afraid of!

0 + 1 = 1

0 x 1 = 0

0 / 1 = 0

1 / 0 is undefined!



The Greeks and a lot of western math thereafter ignored it! There was no zero.

Greek number system

Arabic number	1	2	3	4	5	6	7	8	9
Greek number	α	β	γ	δ	ε	F	ζ	η	θ
Greek name	alpha	beta	gamma	delta	epsilon	digamma	zeta	eta	theta
Sound	a	b	g	d	short e		z	long e	th
Arabic number	10	20	30	40	50	60	70	80	90
Greek number	ι	κ	λ	μ	ν	ξ	ο	π	ϙ
Greek name	iota	kappa	lambda	mu	nu	xi	omicron	pi	koppa
Sound	i	k/c	l	m	n	x	short o	p	
Arabic number	100	200	300	400	500	600	700	800	900
Greek number	ρ	σ	τ	υ	φ	χ	ψ	ω	ϡ
Greek name	rho	sigma	tau	upsilon	phi	chi	psi	omega	sampi
Sound	r	s	t	u	f/ph	ch	ps	long o	

Problem of ignoring zero

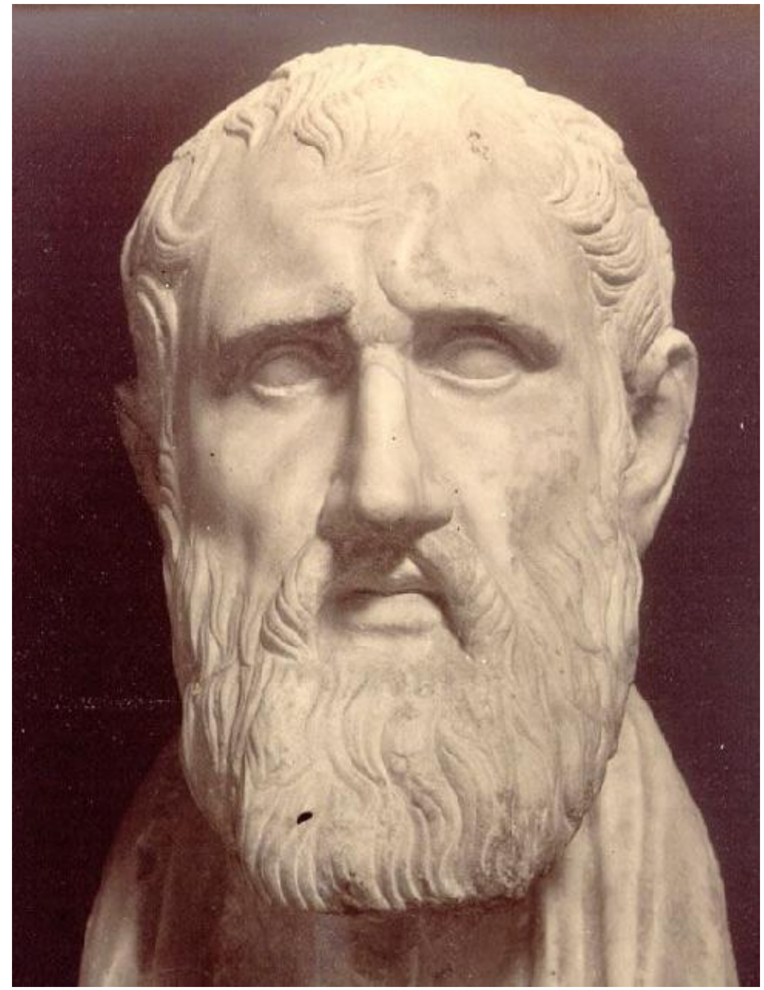
Dionysius's Calendar!

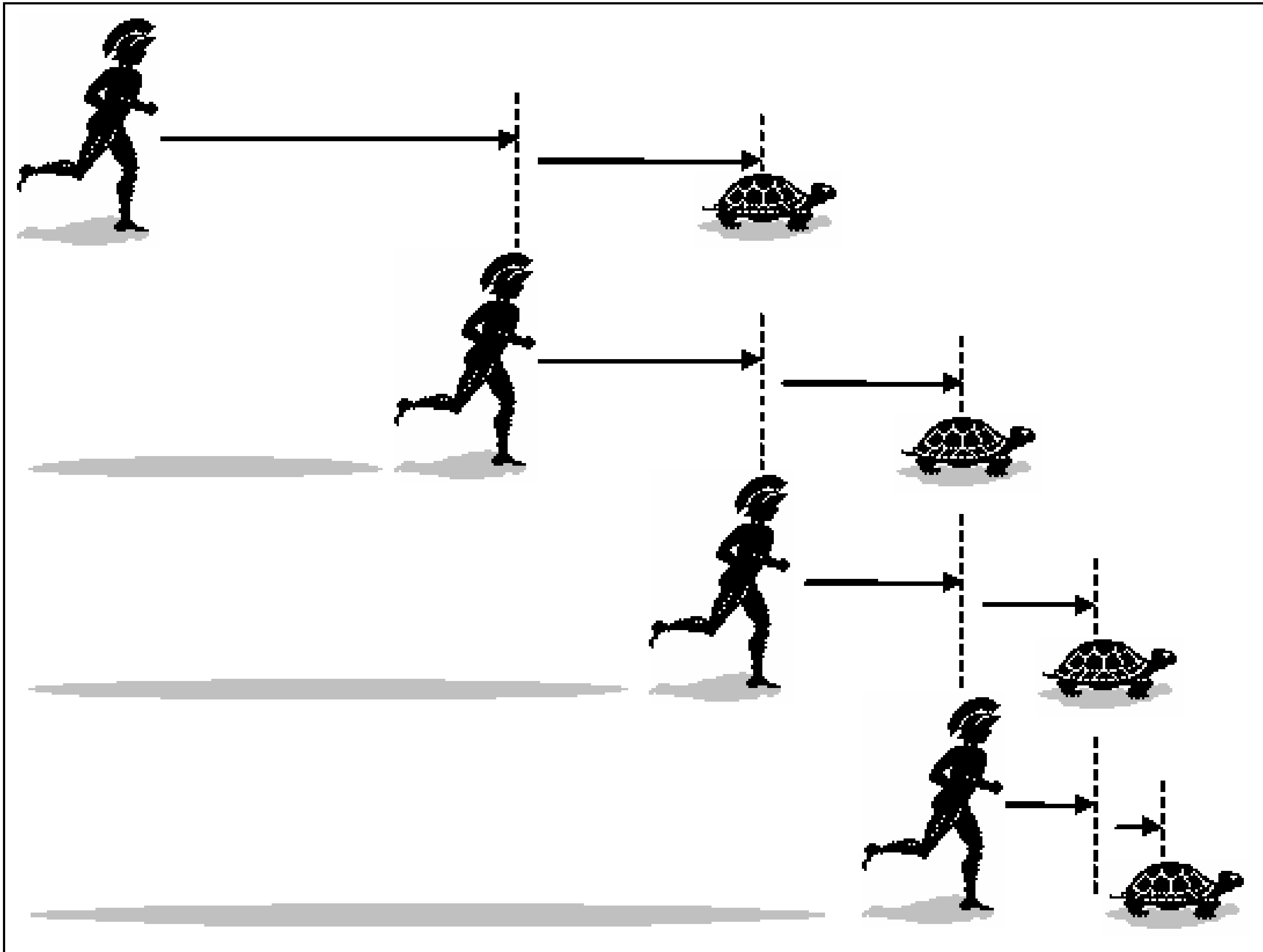
What if you were born in 3 B.C?

2 B.C.	1 B.C.	1 A.D.	2 A.D.
1 year old	2 years old	4 years old	5 years old

More problems from Zeno

Greek philosopher – his paradox Achilles and the tortoise



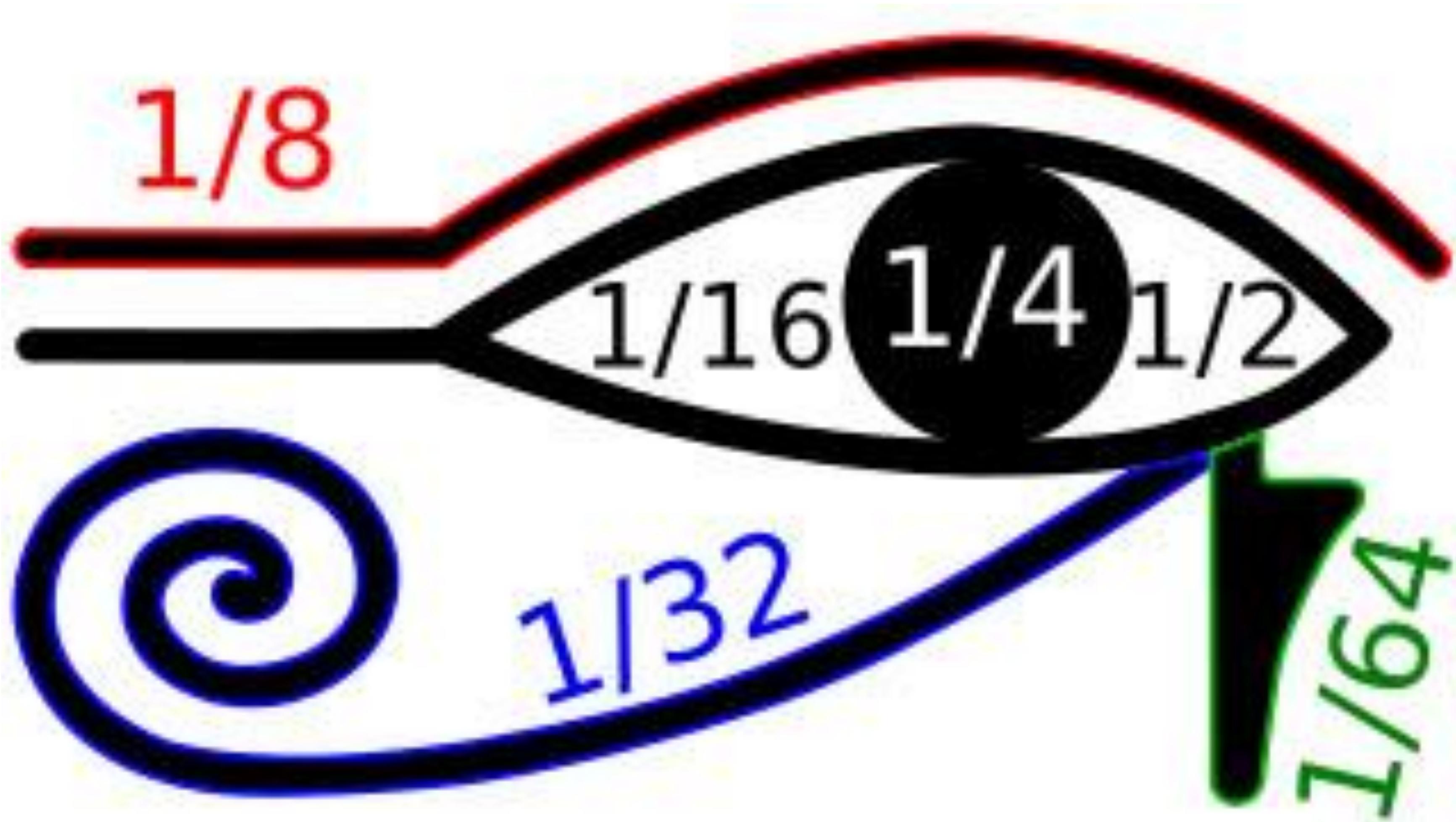


The Eye of Horus

- Not ‘Only Connect’ but an aide memoire for fractions.

http://io9.com/5907604/this-well-known-egyptian-symbol-is-actually-an-early-math-problem

The Eye of Horus – Egyptian Fractions



See “A Curious History of Mathematics by Joel Levy (P. 25)