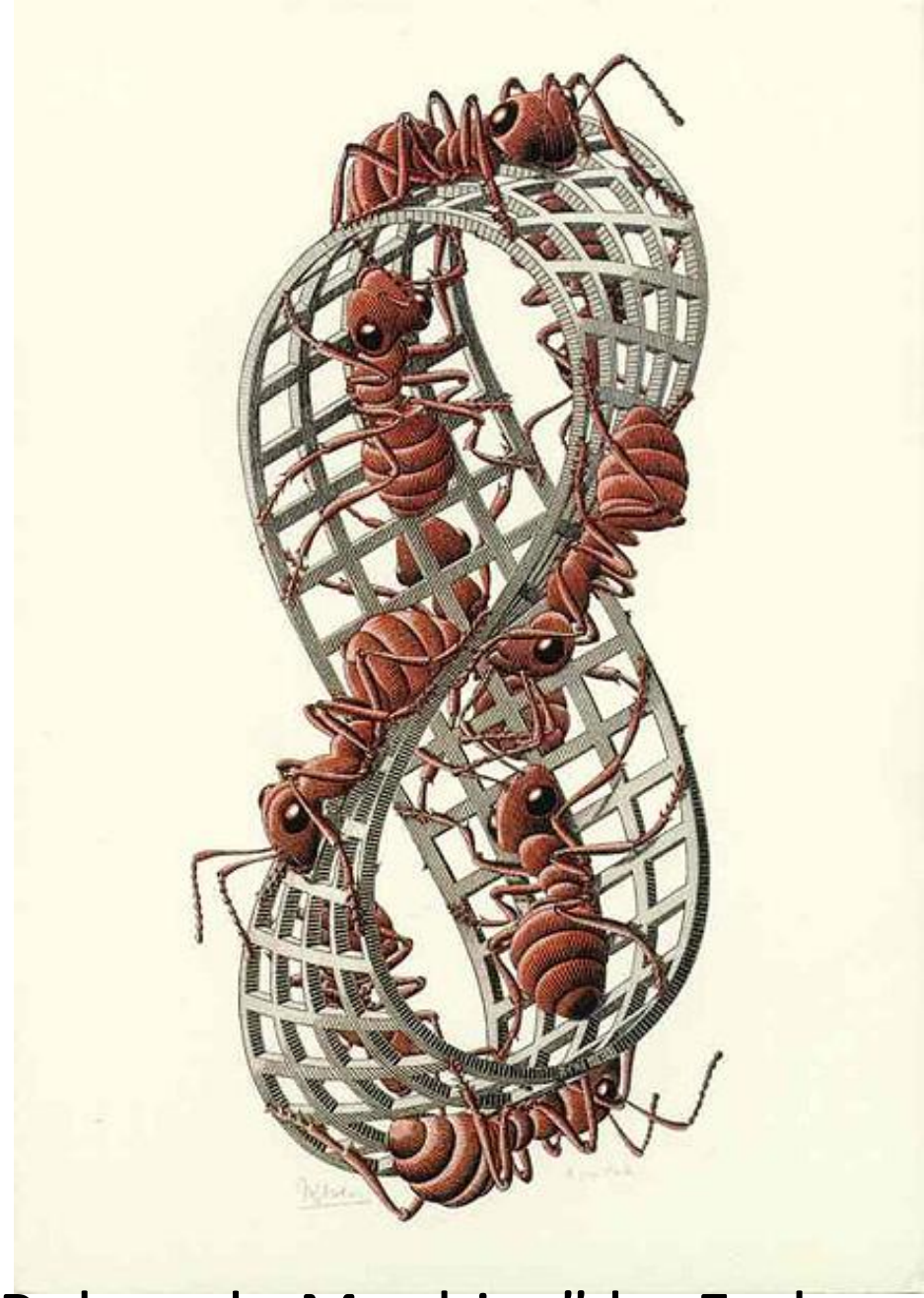
The Joy of Mathematics - 4

a trip through the dimensions without a Tardis

Last week

There are some strange things that already lurk in our visible world of 3 dimensions, but what lies beyond?



“Ruban de Moebius” by Escher

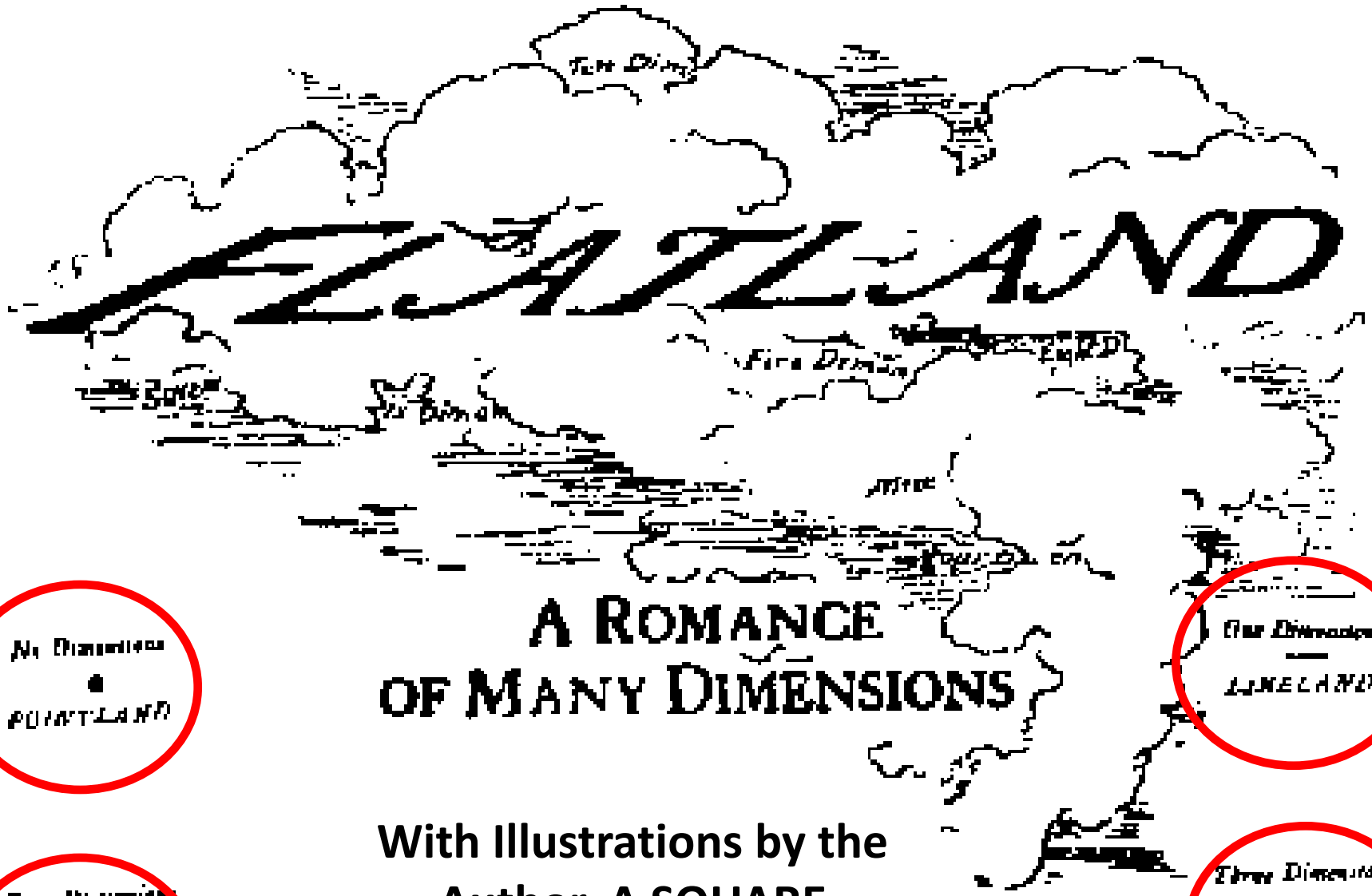
What would the world of the fourth spacial dimension look like

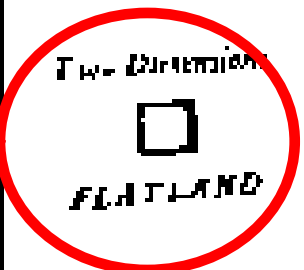
Let's start by looking at what it might be like trying to imagine what it might be like to live in a 2-D world like "Flatland."

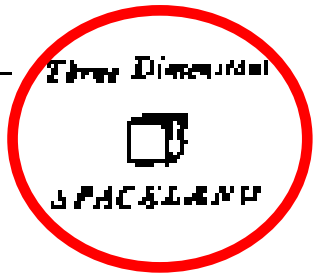
What are the dimensions....

- 0 dimensions
  - A point in our space with no size whatsoever
- 1 dimension
  - A line without any thickness and finite length
- 2 dimensions
  - A finite plane without any thickness – shape?
- 3 dimensions
  - What we think we know! We live there!

"O day and night, but this is wondrous strange"







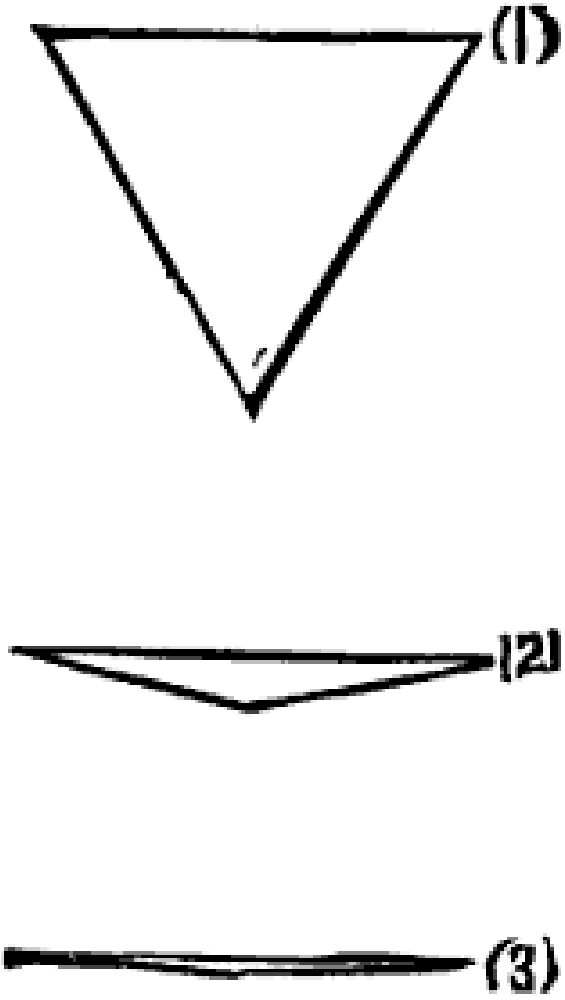
With Illustrations by the Author, A SQUARE (Edwin A. Abbott 1838-1926)

To
The Inhabitants of SPACE IN GENERAL
And H. C. IN PARTICULAR
This Work is Dedicated
By a Humble Native of Flatland
In the Hope that
Even as he was Initiated into the Mysteries
Of THREE Dimensions
Having been previously conversant
With ONLY TWO
So the Citizens of that Celestial Region
May aspire yet higher and higher
To the Secrets of FOUR FIVE OR EVEN SIX Dimensions
Thereby contributing
To the Enlargement of THE IMAGINATION
And the possible Development
Of that most rare and excellent Gift of MODESTY
Among the Superior Races
Of SOLID HUMANITY

A matter or perception

- Triangle turning into a line

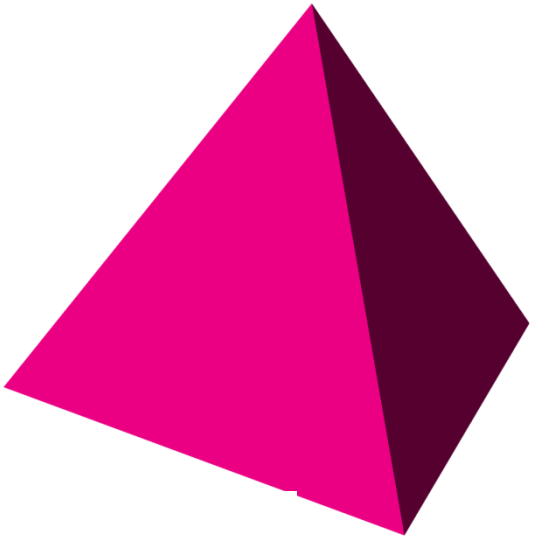
In Flatland

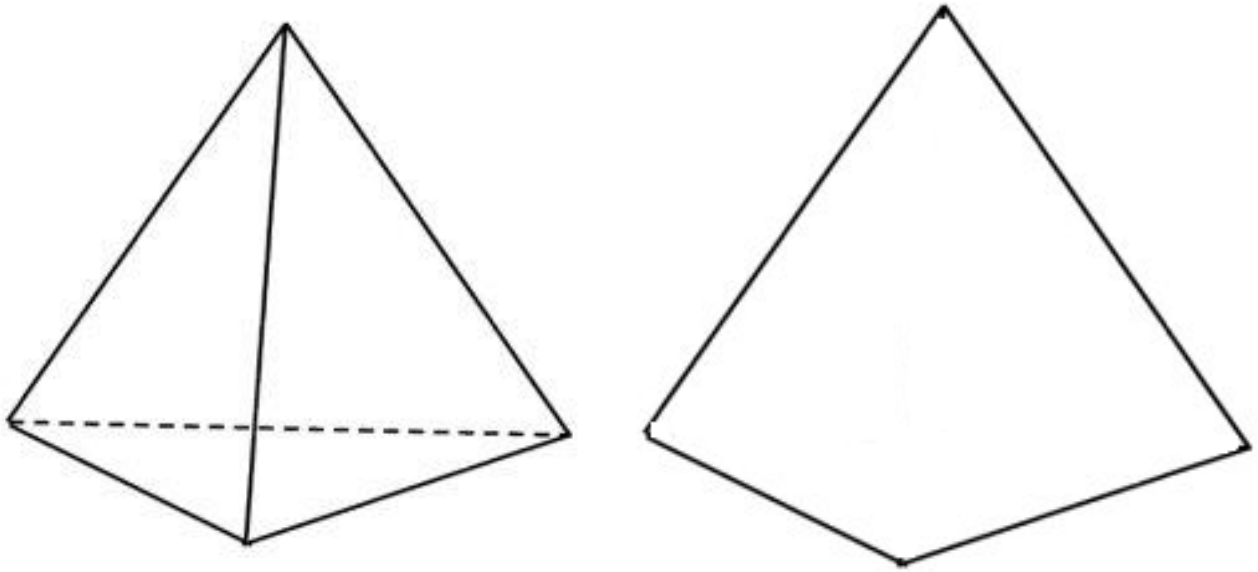


A matter or perception

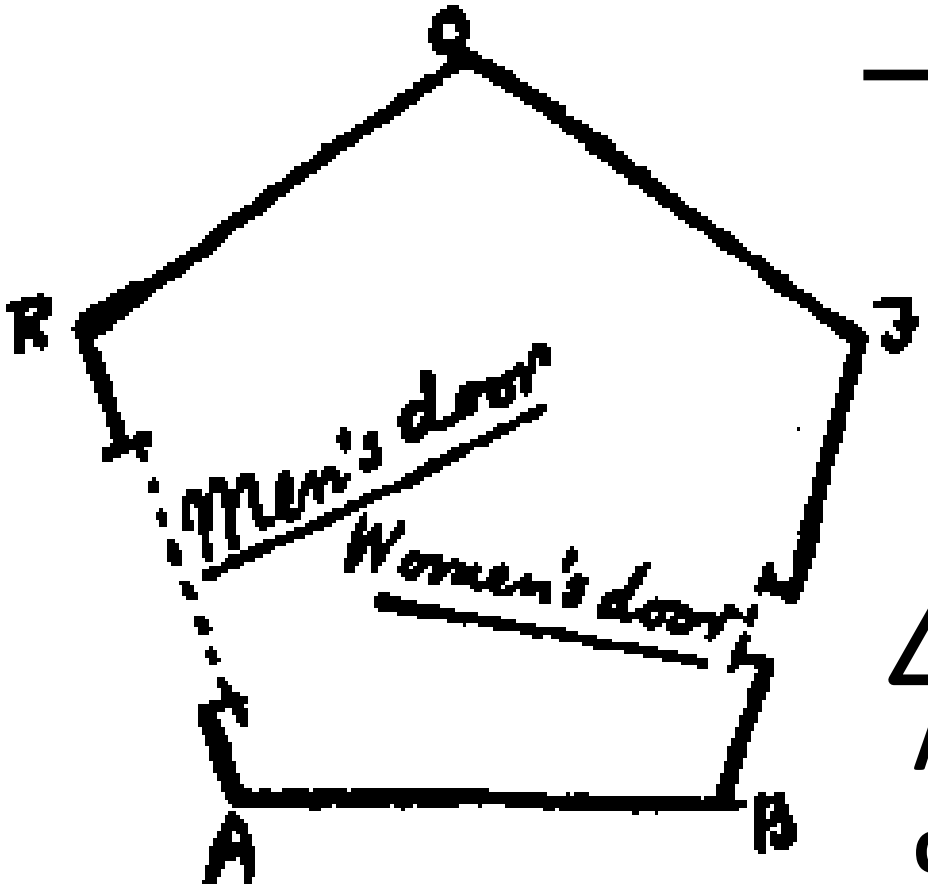
A tetrahedron

But what does
it look like if you ignore
the shading?

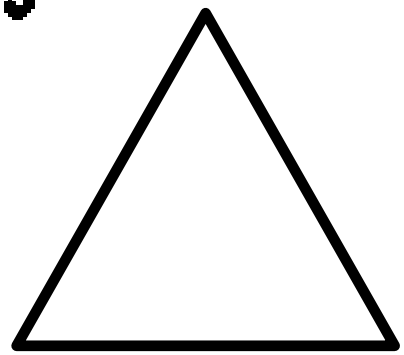




Flatland entities

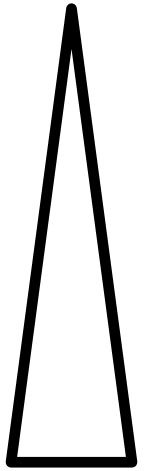


A woman



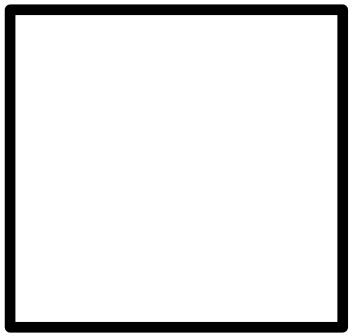
A middle-class man

Equilateral



A working class man

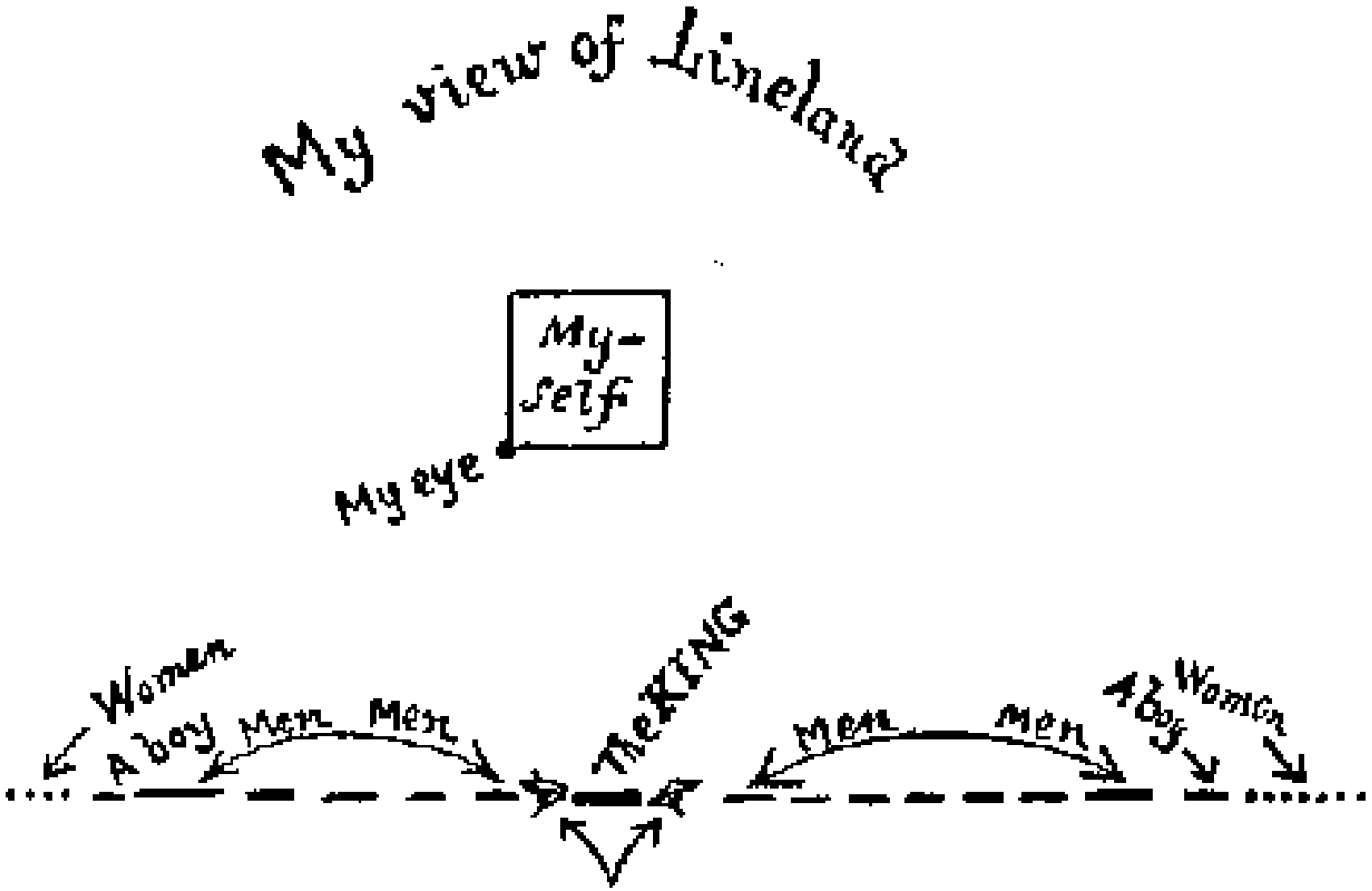
Isosceles



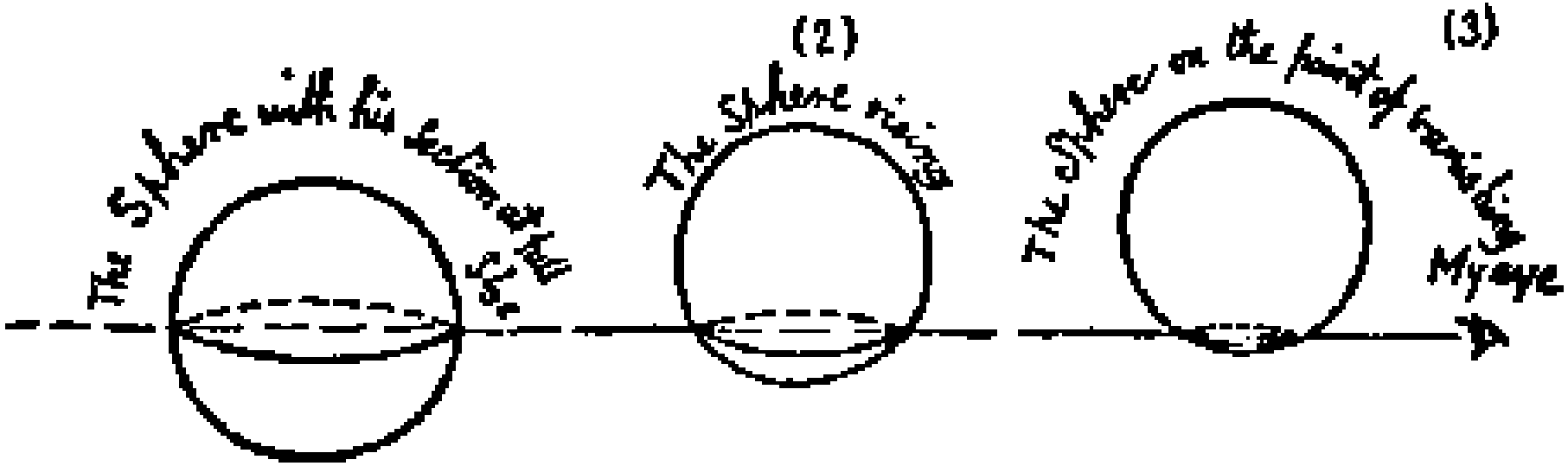
Professional men and Gentlemen

Women! To be feared!

_____


My view of Lineland
My-self
My eye
Women
A boy
Men
Men
The KING
Men
Men
A boy
Women
The KING's eyes

A Spaceman comes to Flatland



“You cannot indeed see more than one of my sections, or Circles, at a time; for you have no power to raise your eye out of the plane of Flatland; but you can at least see that, as I rise in Space, so my sections become smaller. See now, I will rise; and the effect upon your eye will be that my Circle will become smaller and smaller till it dwindles to a point and finally vanishes. There was no "rising" that I could see; but he diminished and finally vanished.”

An obscene object – the Cube

Sphere. “Exactly. Then you see you have answered your own question. The Cube which you will generate will be bounded by six sides, that is to say, six of your insides. You see it all now, eh?”

"Monster," I shrieked, "be thou juggler, enchanter, dream, or devil, no more will I endure thy mockeries. Either thou or I must perish."

He leaves Flatland for Spaceland

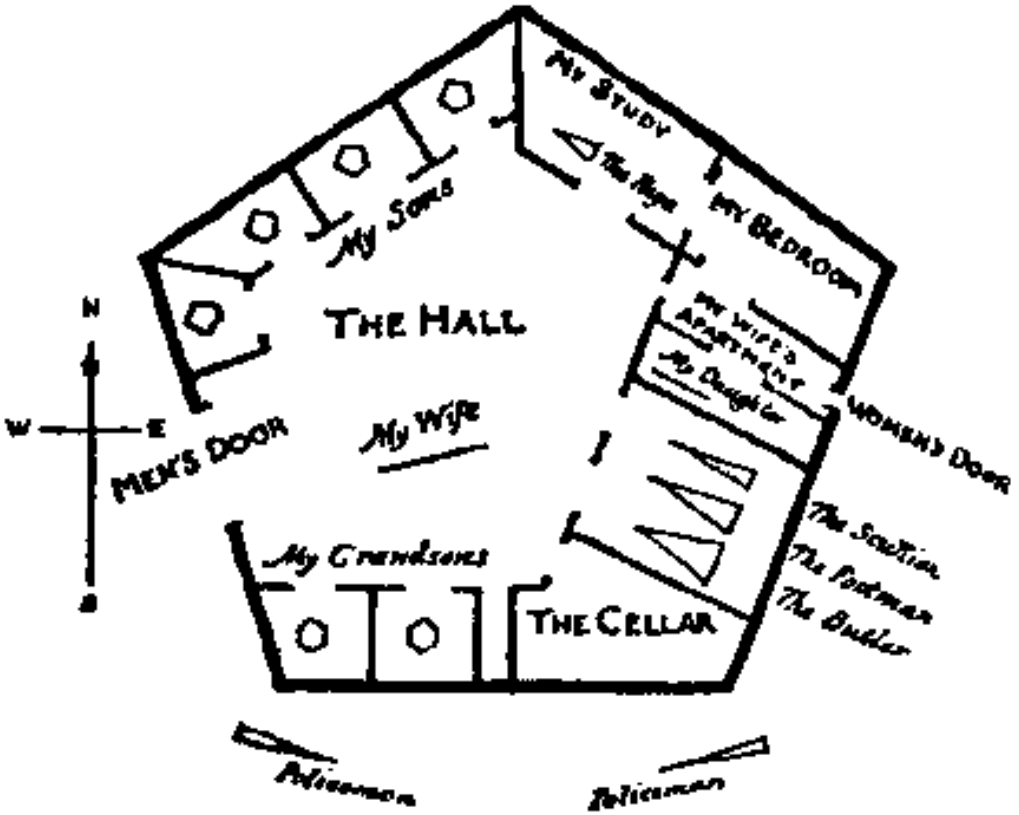
I shrieked aloud in agony, "Either this is madness or it is Hell.“

"It is neither," calmly replied the voice of the Sphere, "it is Knowledge; it is Three Dimensions: open your eye once again and try to look steadily.“

A different viewpoint!

"I looked below, and saw with my physical eye all that domestic individuality which I had hitherto merely inferred with the understanding. And how poor and shadowy was the inferred conjecture in comparison with the reality which I now beheld! .....

.....I could discern even the contents of my cabinet, and the two chests of gold and the tablets of which the sphere had made mention."



Into the 4th Dimension....

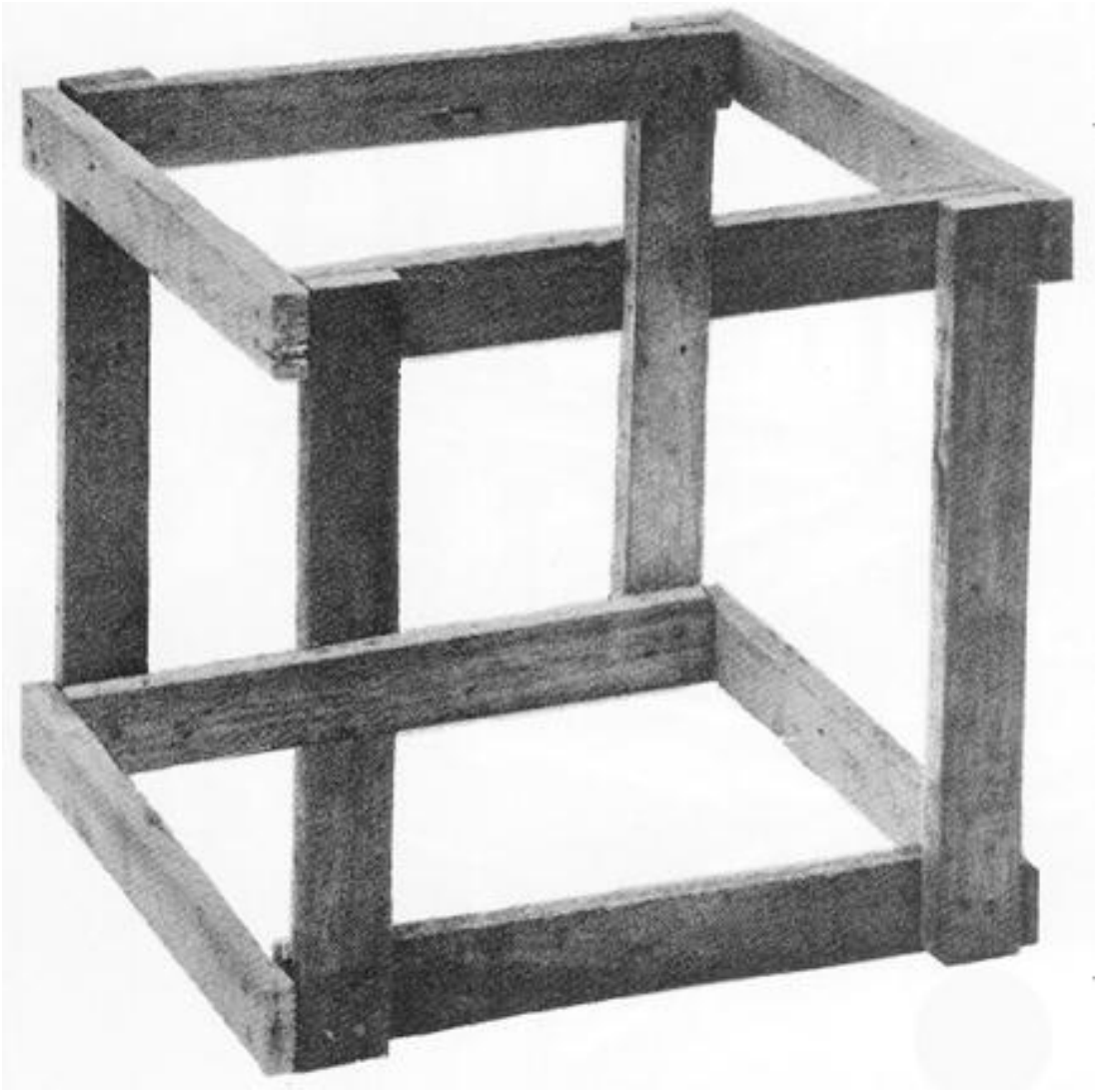
- “In One Dimension, did not a moving Point produce a Line with two terminal points?
- In Two Dimensions, did not a moving Line produce a Square with four terminal points?
- In Three Dimensions, did not a moving Square produce - did not this eye of mine behold it - that blessed Being, a Cube, with eight terminal points?
- And in Four Dimensions shall not a moving Cube - alas, for Analogy, and alas for the Progress of Truth, if it be not so - shall not, I say, the motion of a divine Cube result in a still more divine Organization with sixteen terminal points?”

(keep doubling the number of vertices)

Flatland the film: http://www.youtube.com/watch?v=Mfglluny8Z0

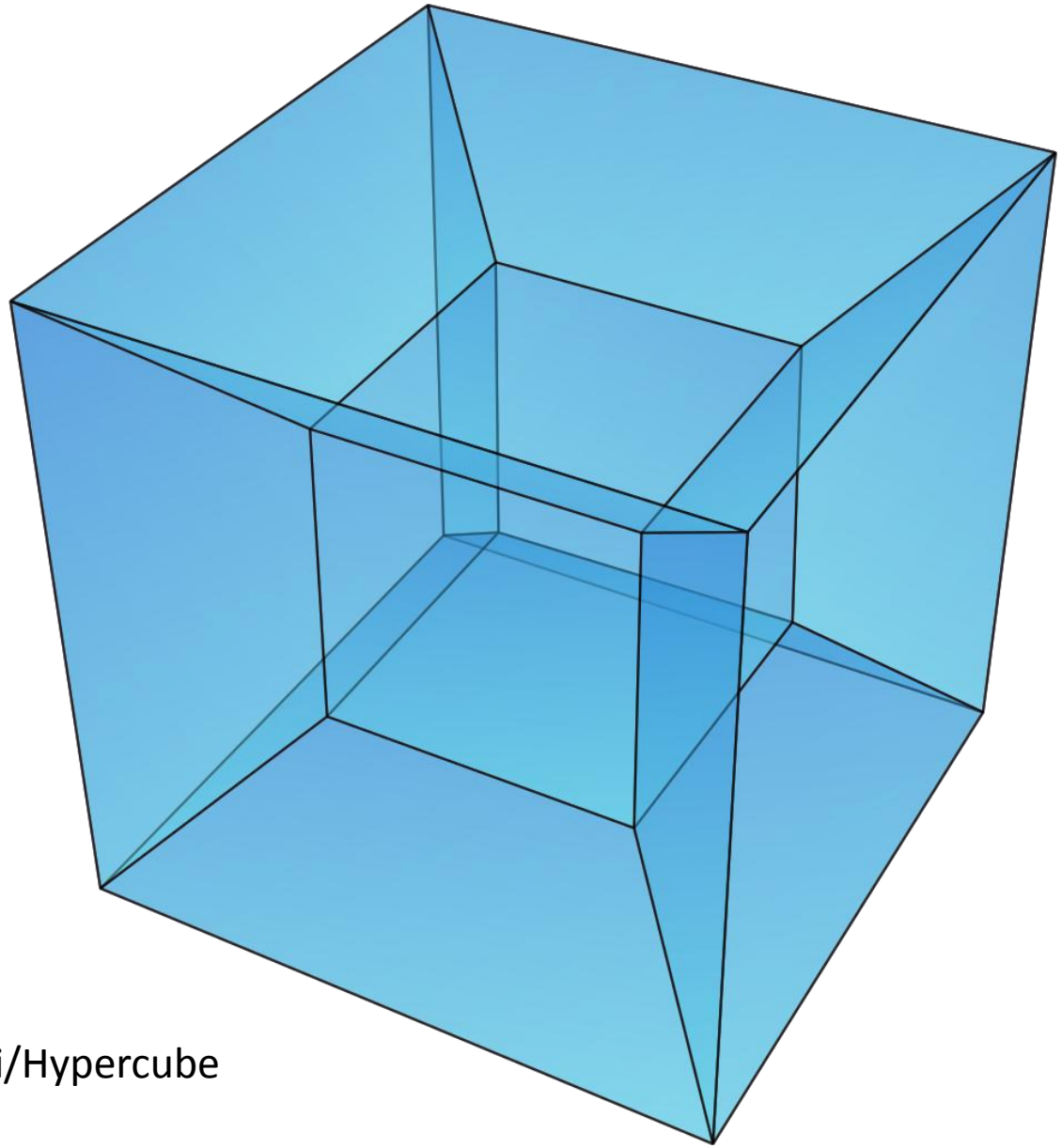
Not a cube! 2d deceives the eye!

Escher again!



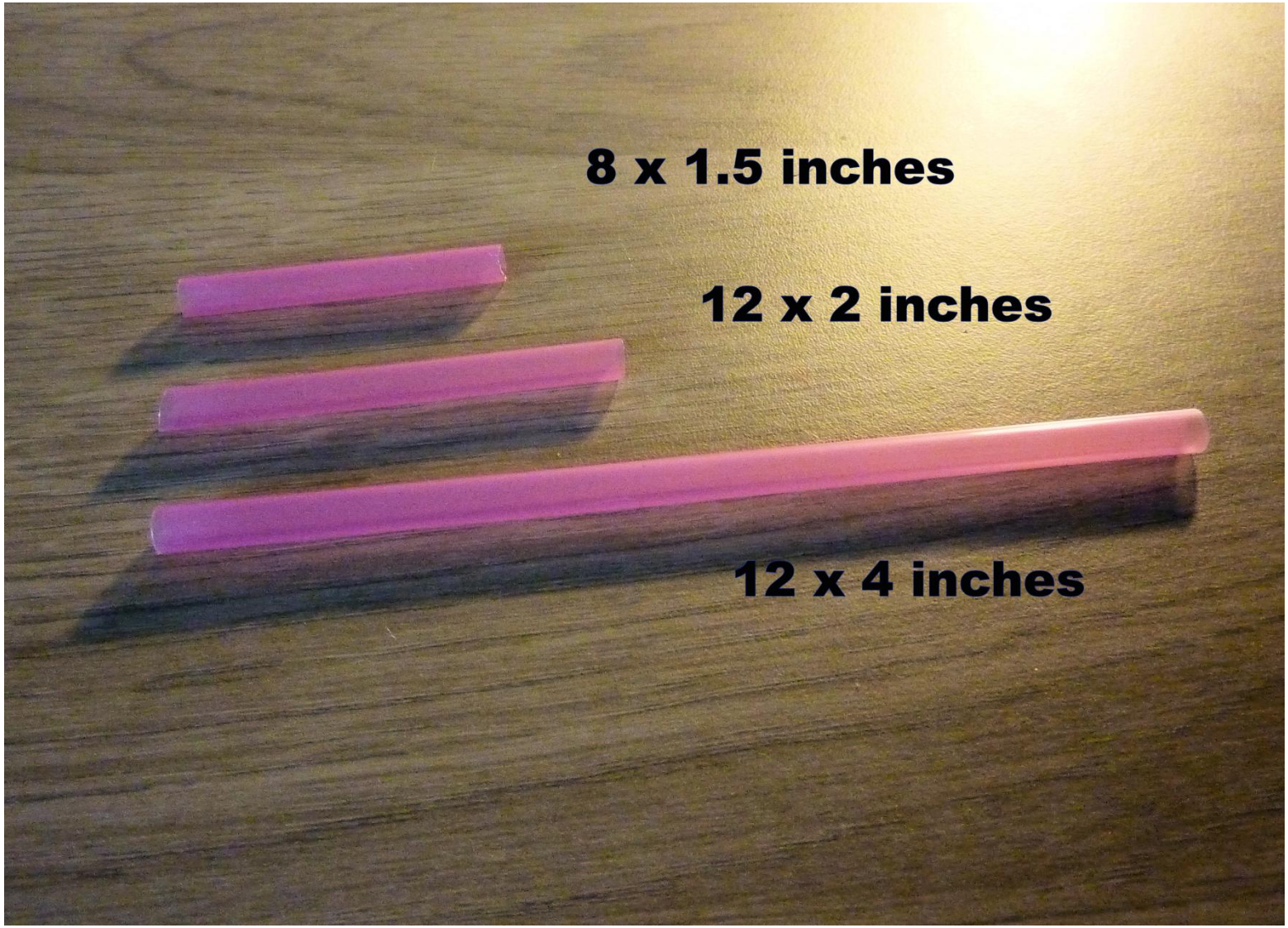
The Hypercube or Tesseract

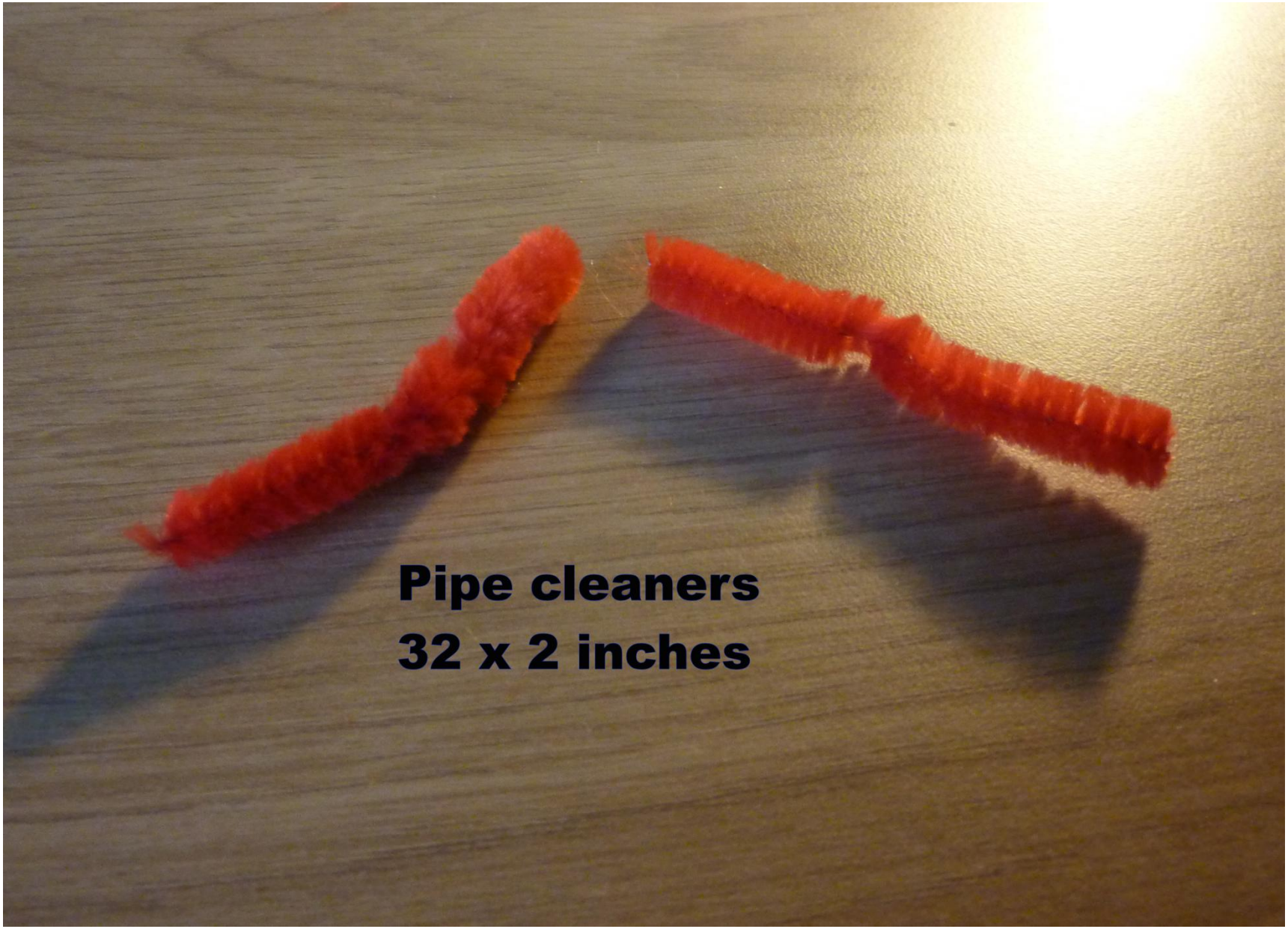
Here is a two dimensional projection of a four dimensional cube:

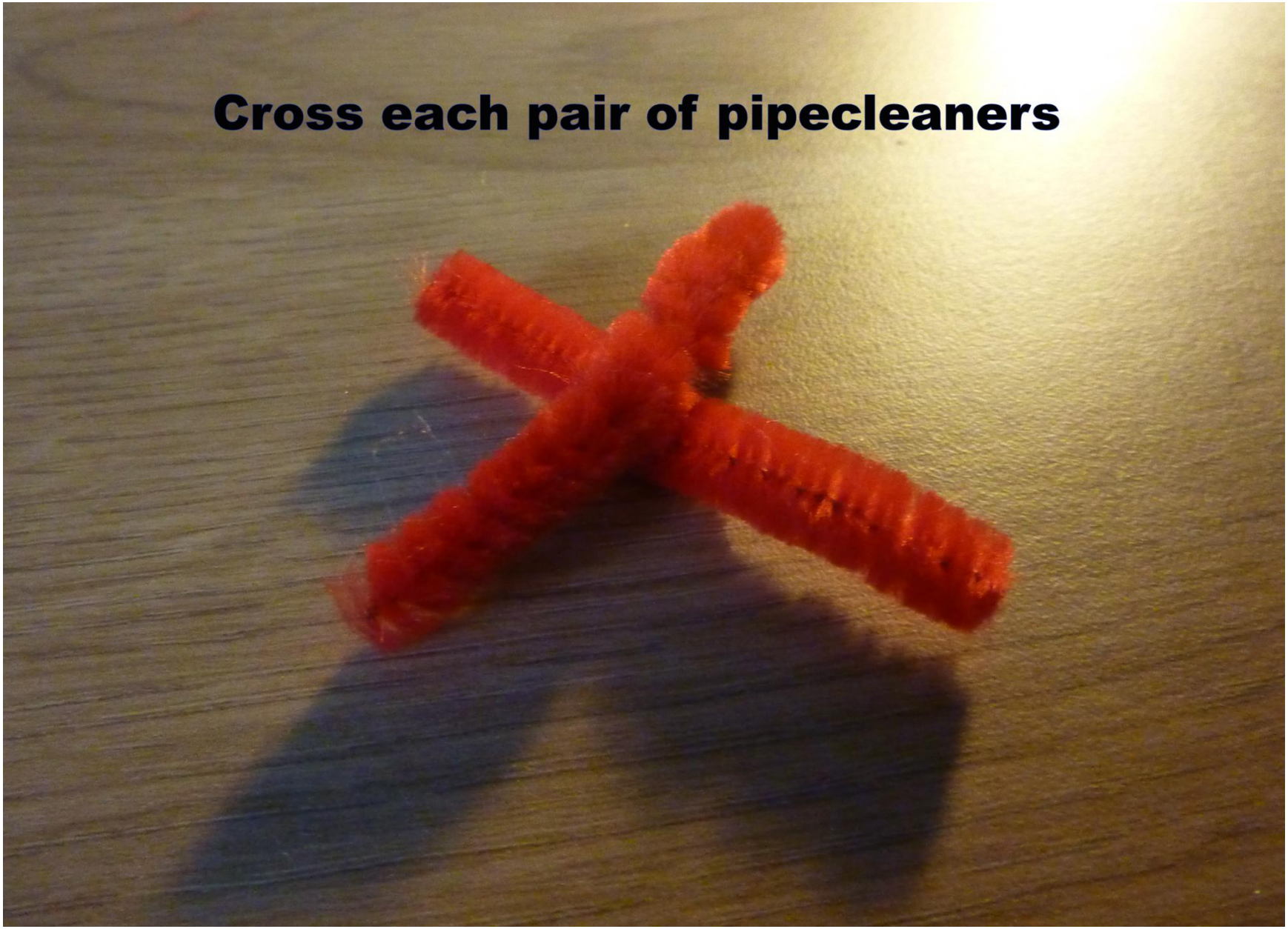


http://en.wikipedia.org/wiki/Hypercube

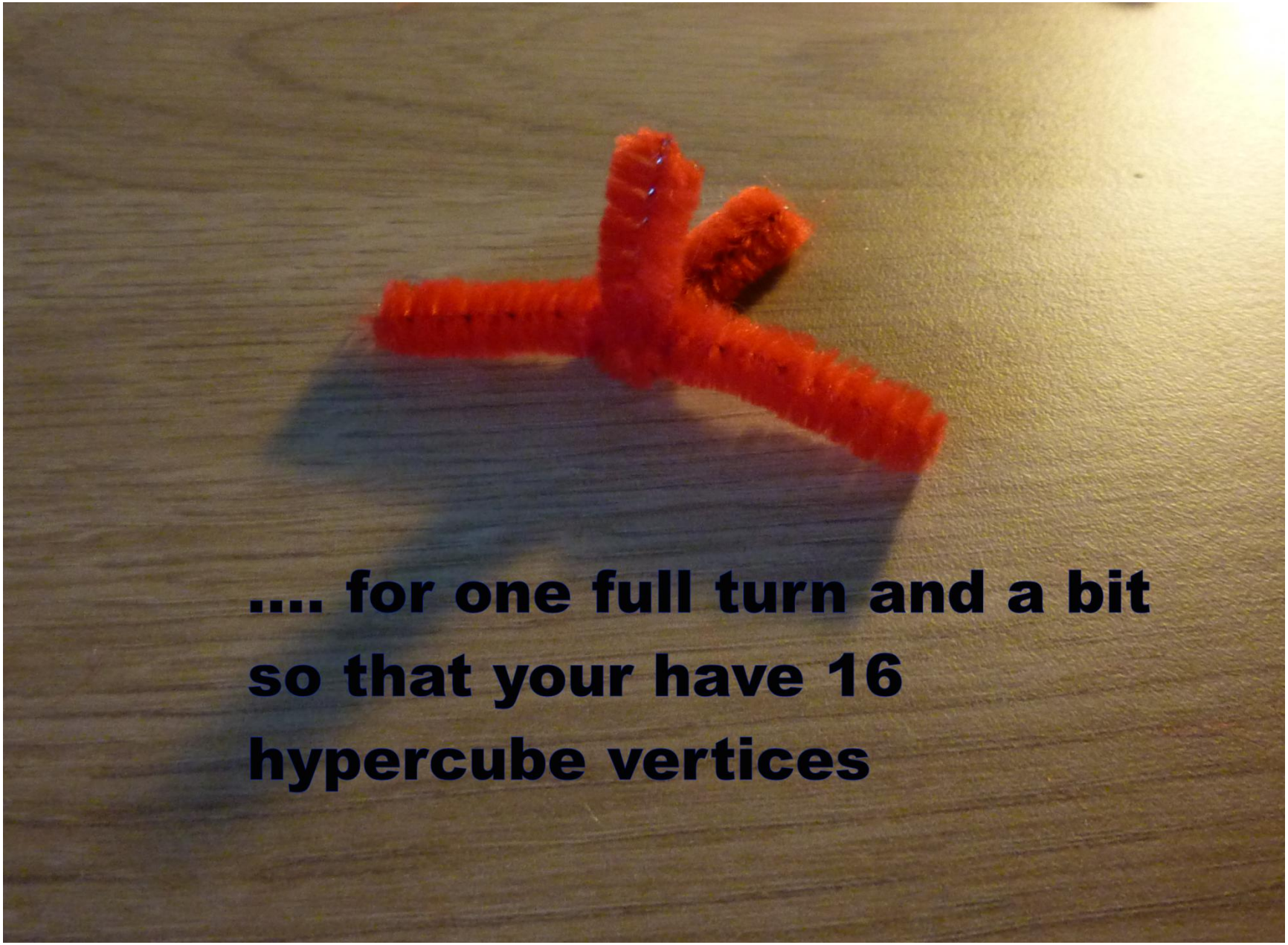
Making a 3D Hypercube model with straws and pipe cleaners



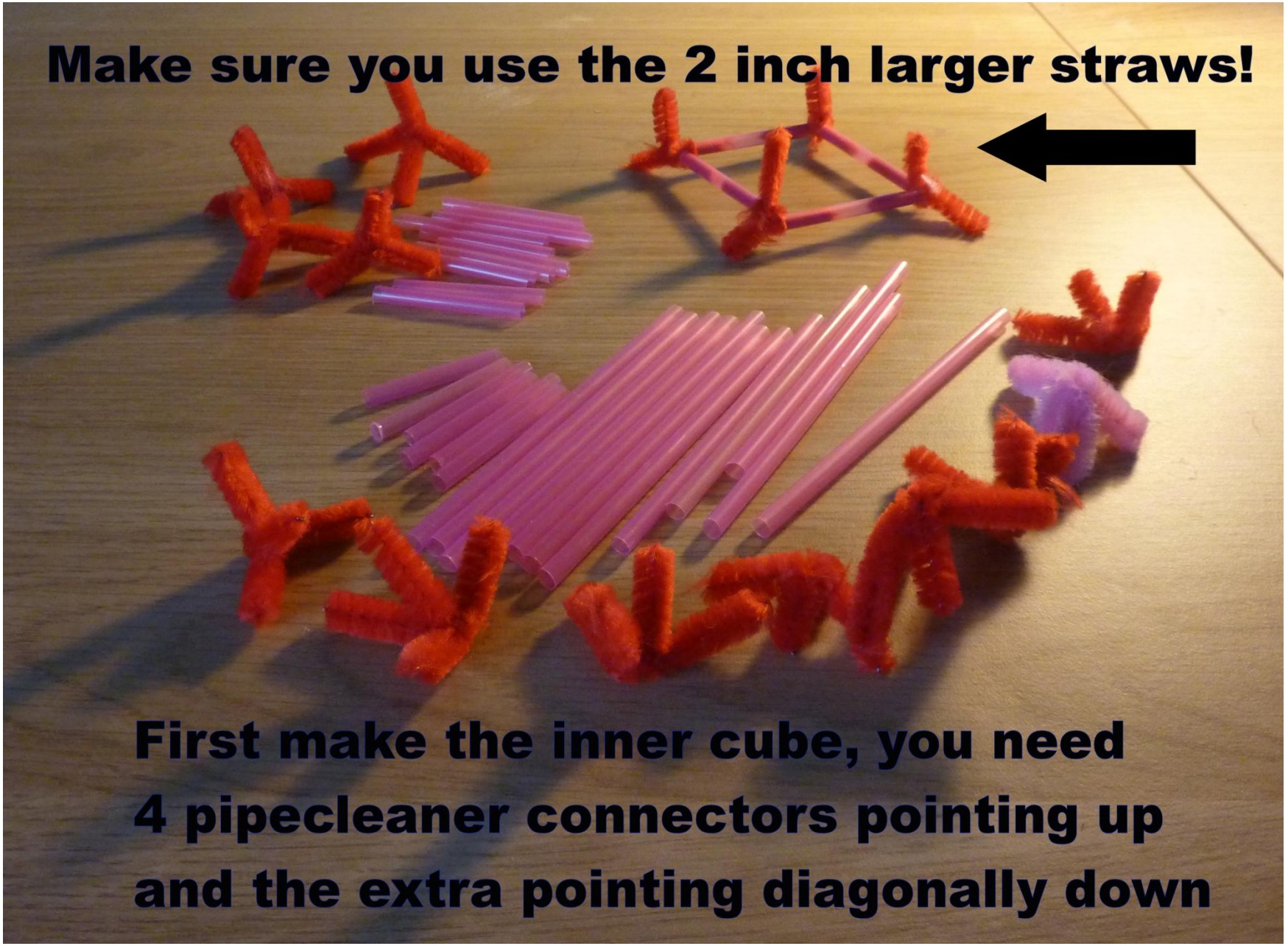

Pipe cleaners
32 x 2 inches

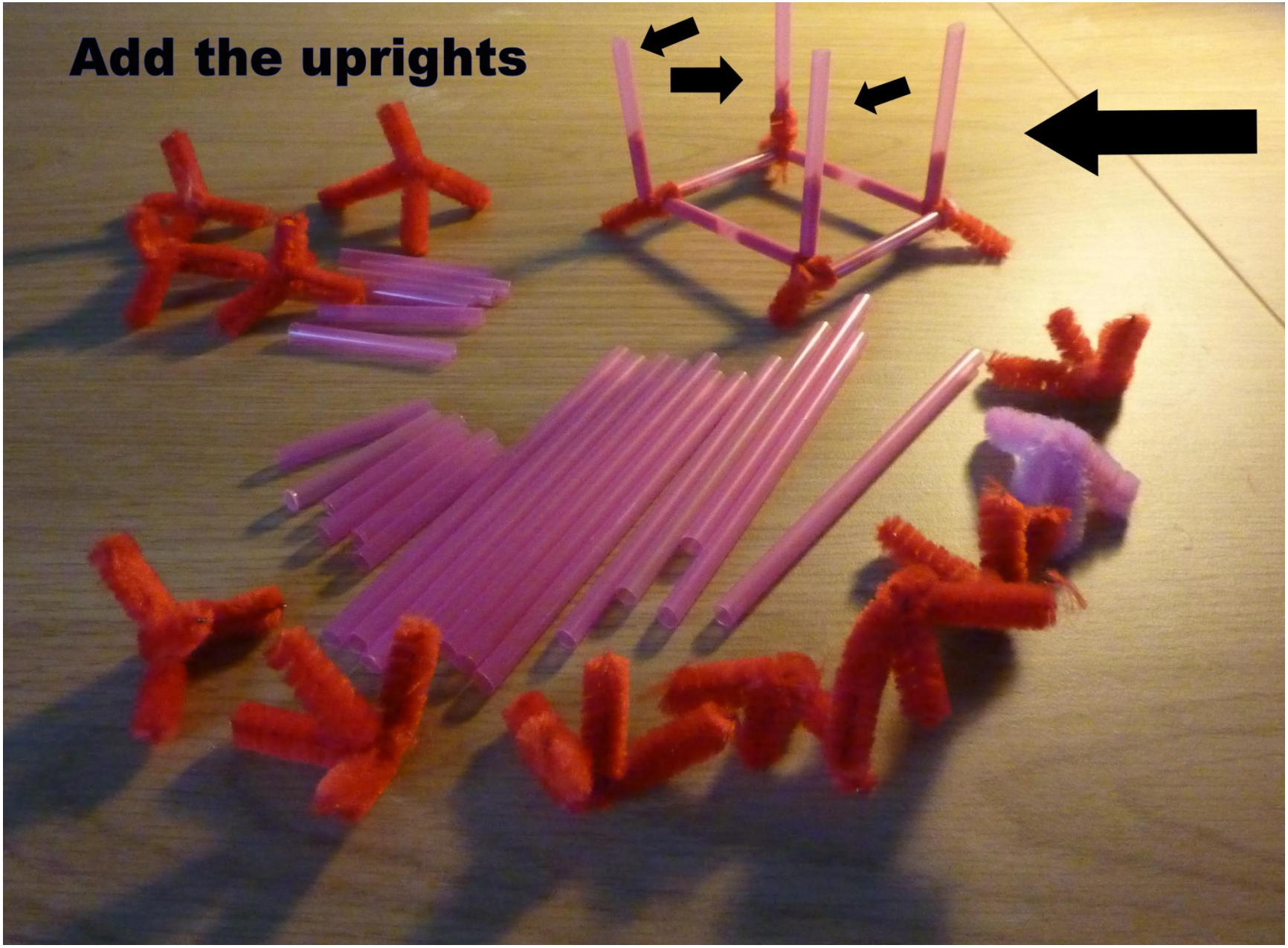

Cross each pair of pipecleaners

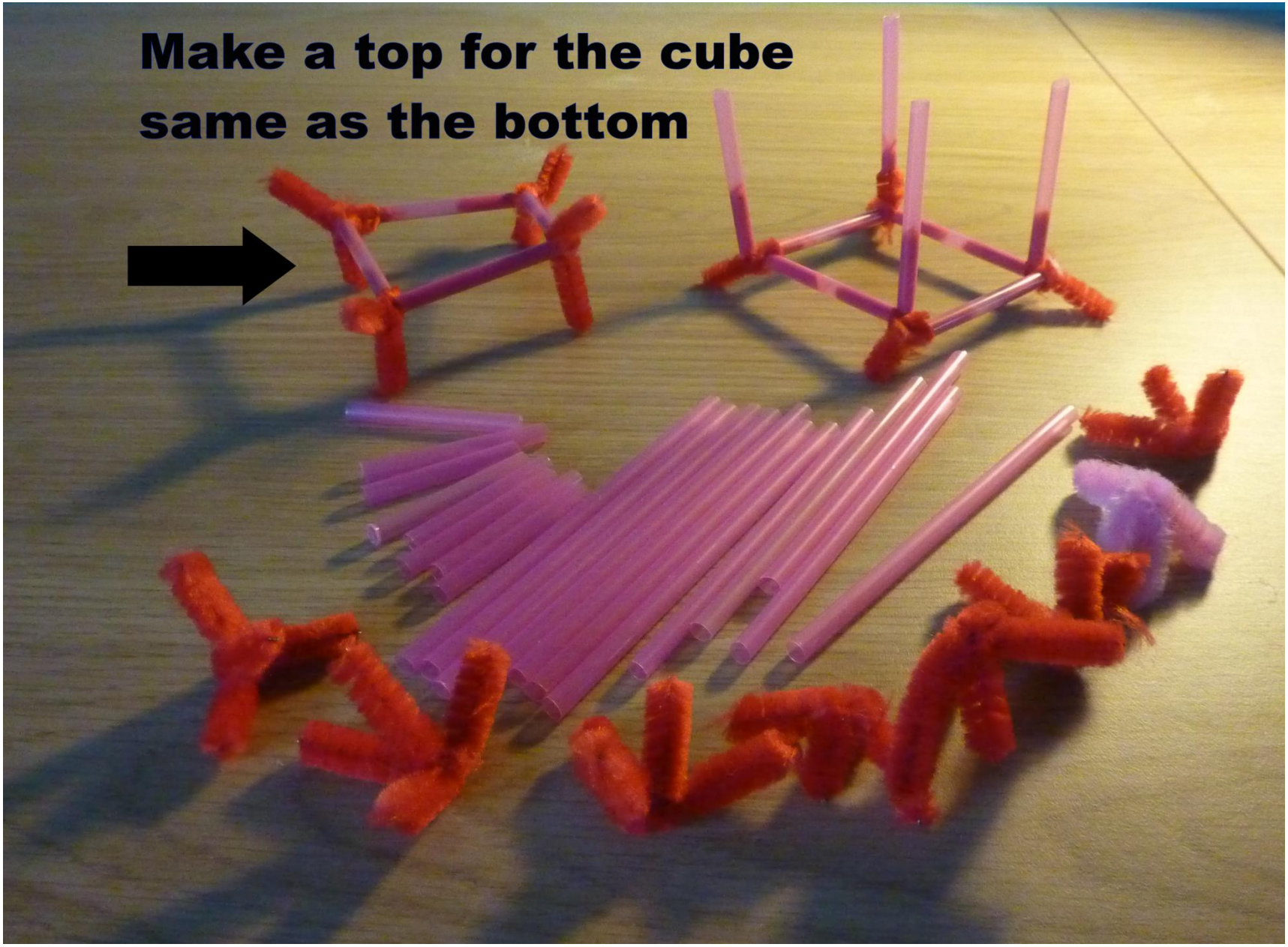

Twist.....

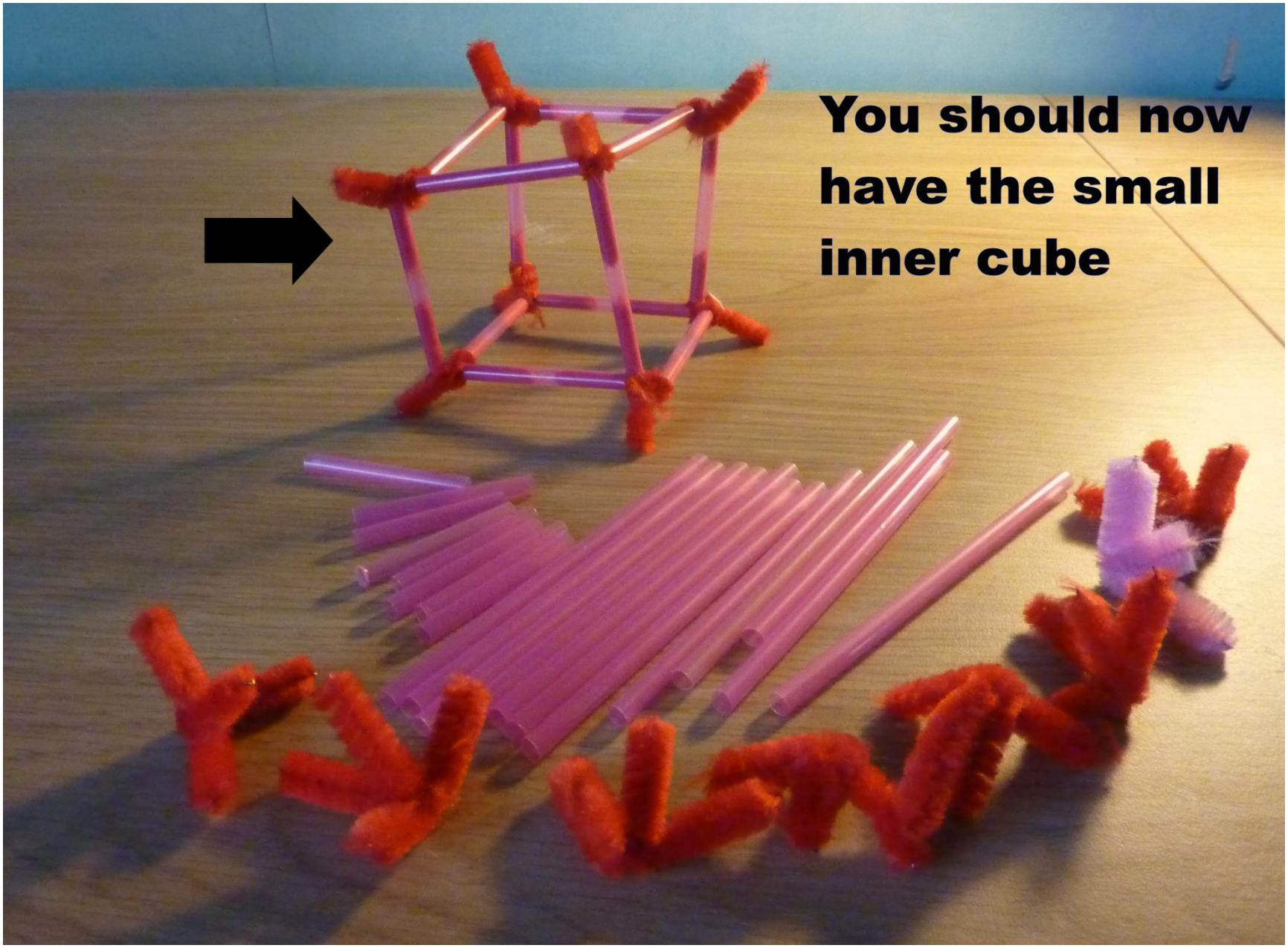

.... for one full turn and a bit
so that your have 16
hypercube vertices

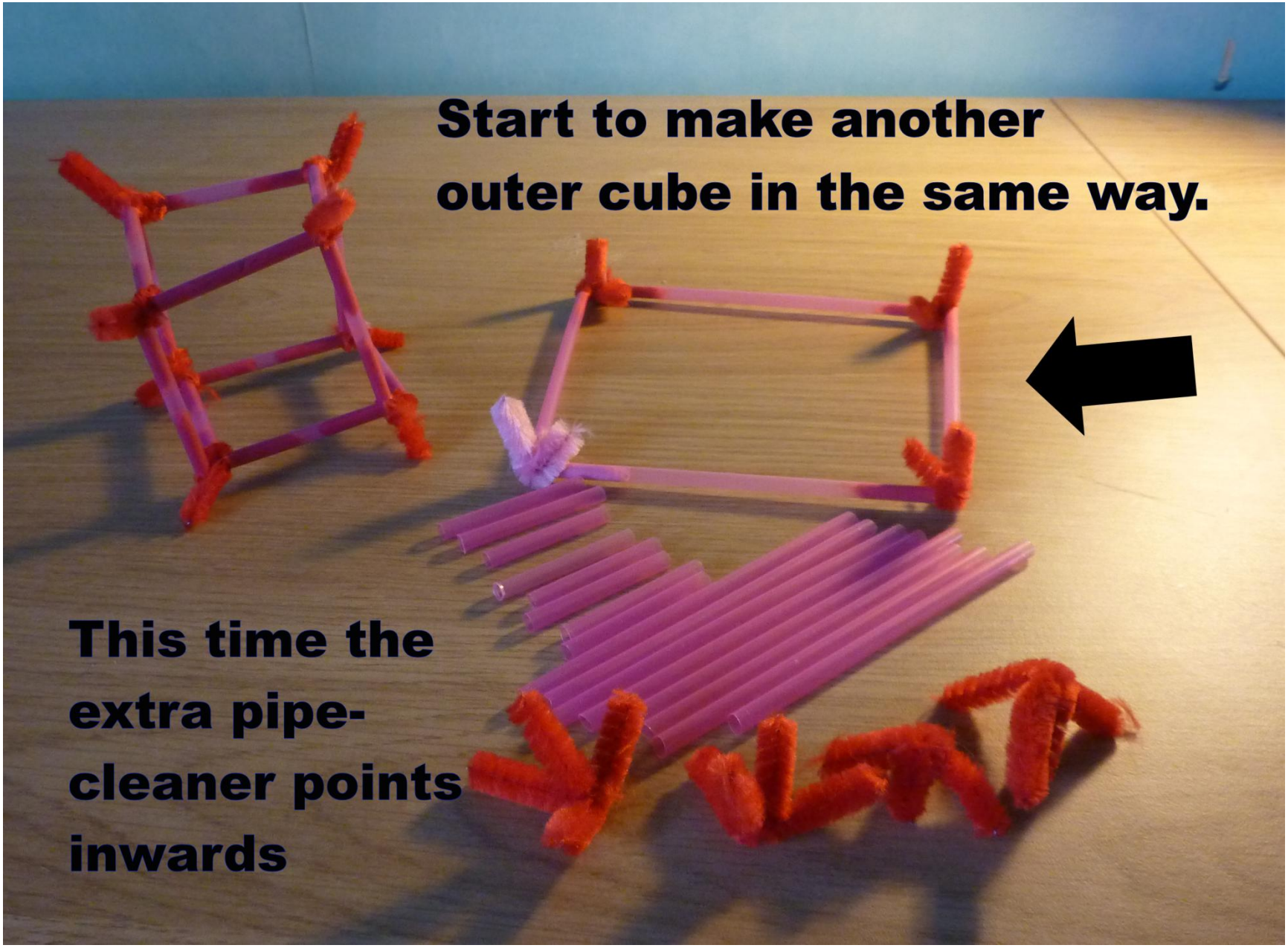

This is now you hypercube kit
ready to assemble

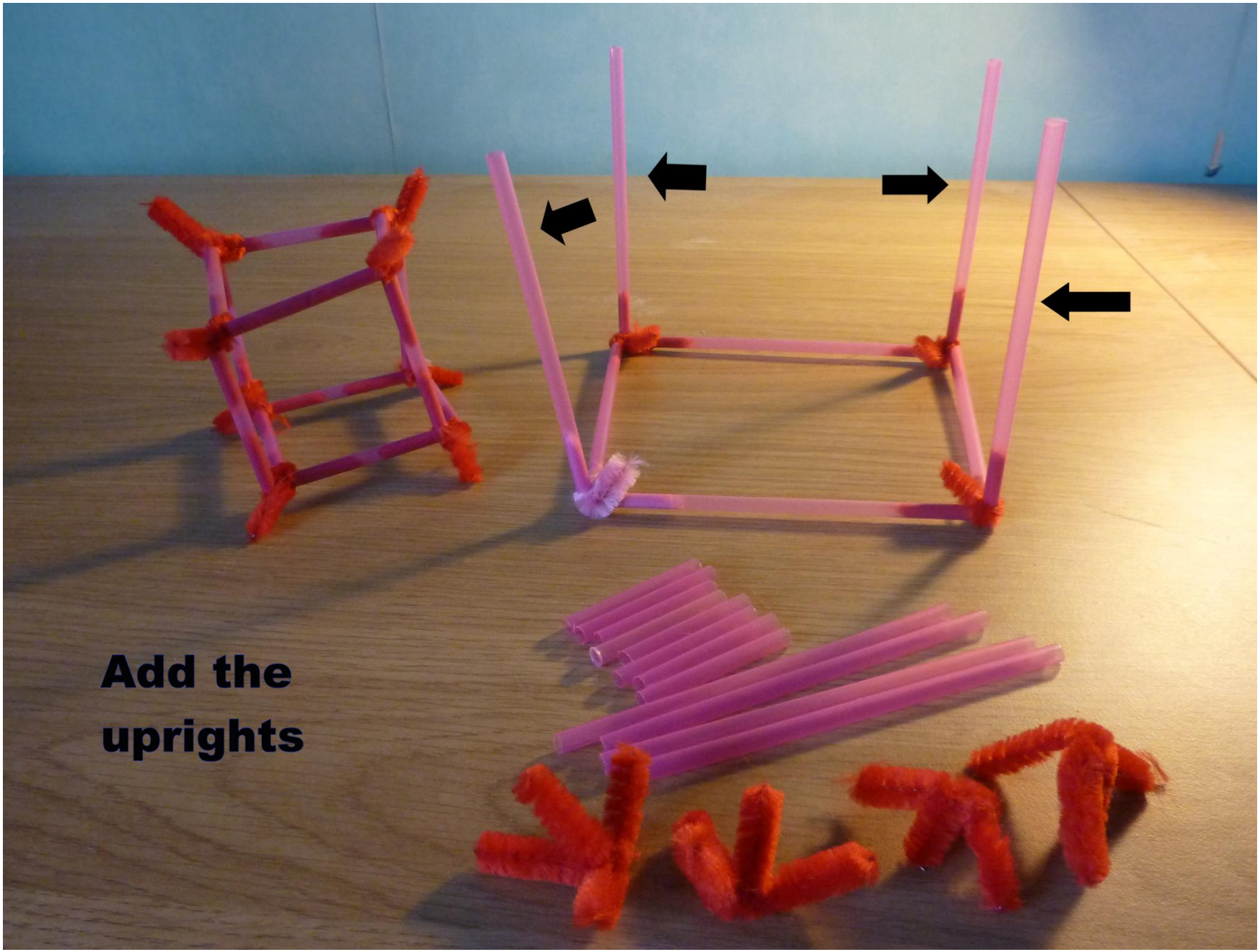

Make sure you use the 2 inch larger straws!
First make the inner cube, you need
4 pipecleaner connectors pointing up
and the extra pointing diagonally down

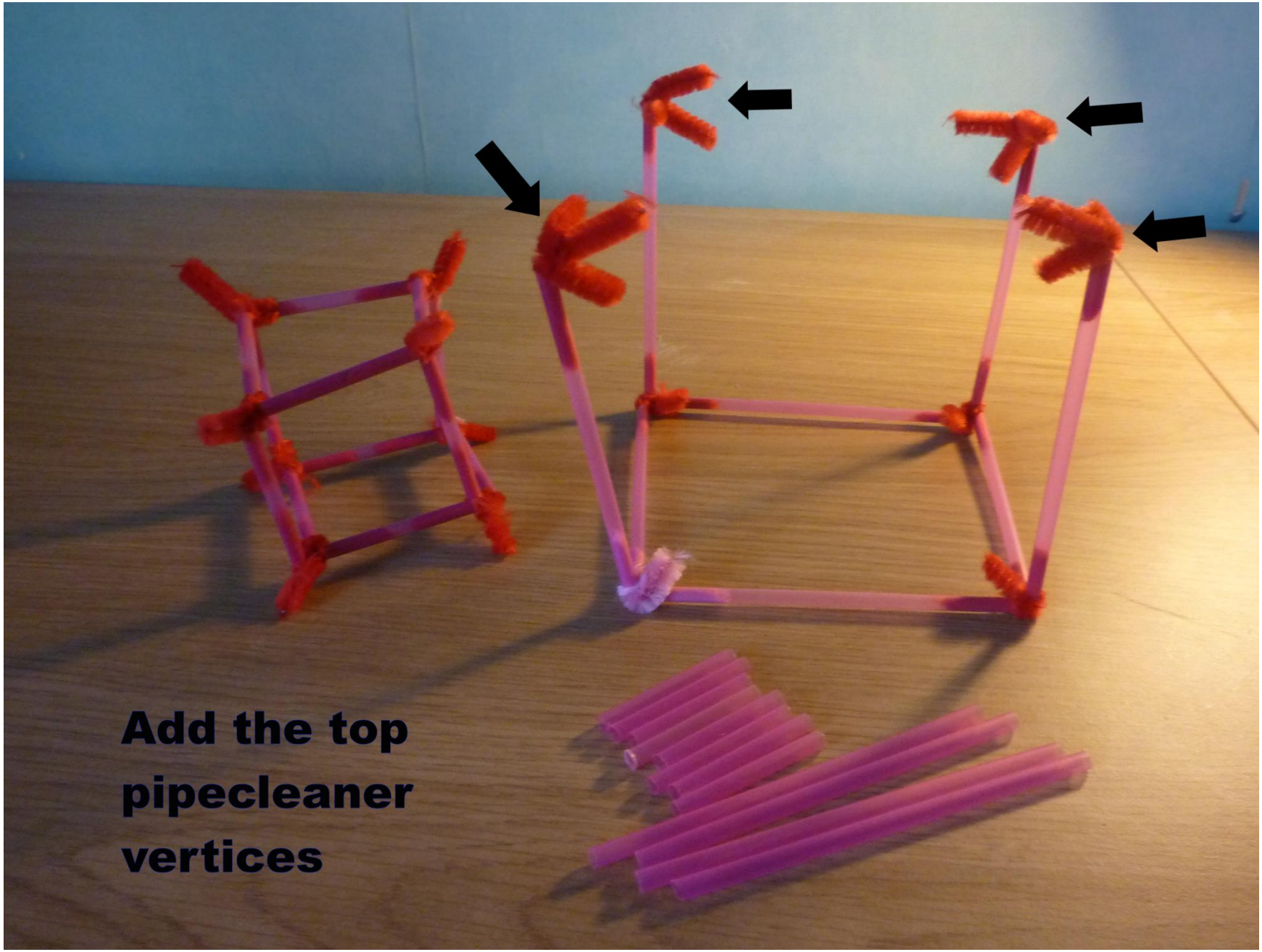

Add the uprights

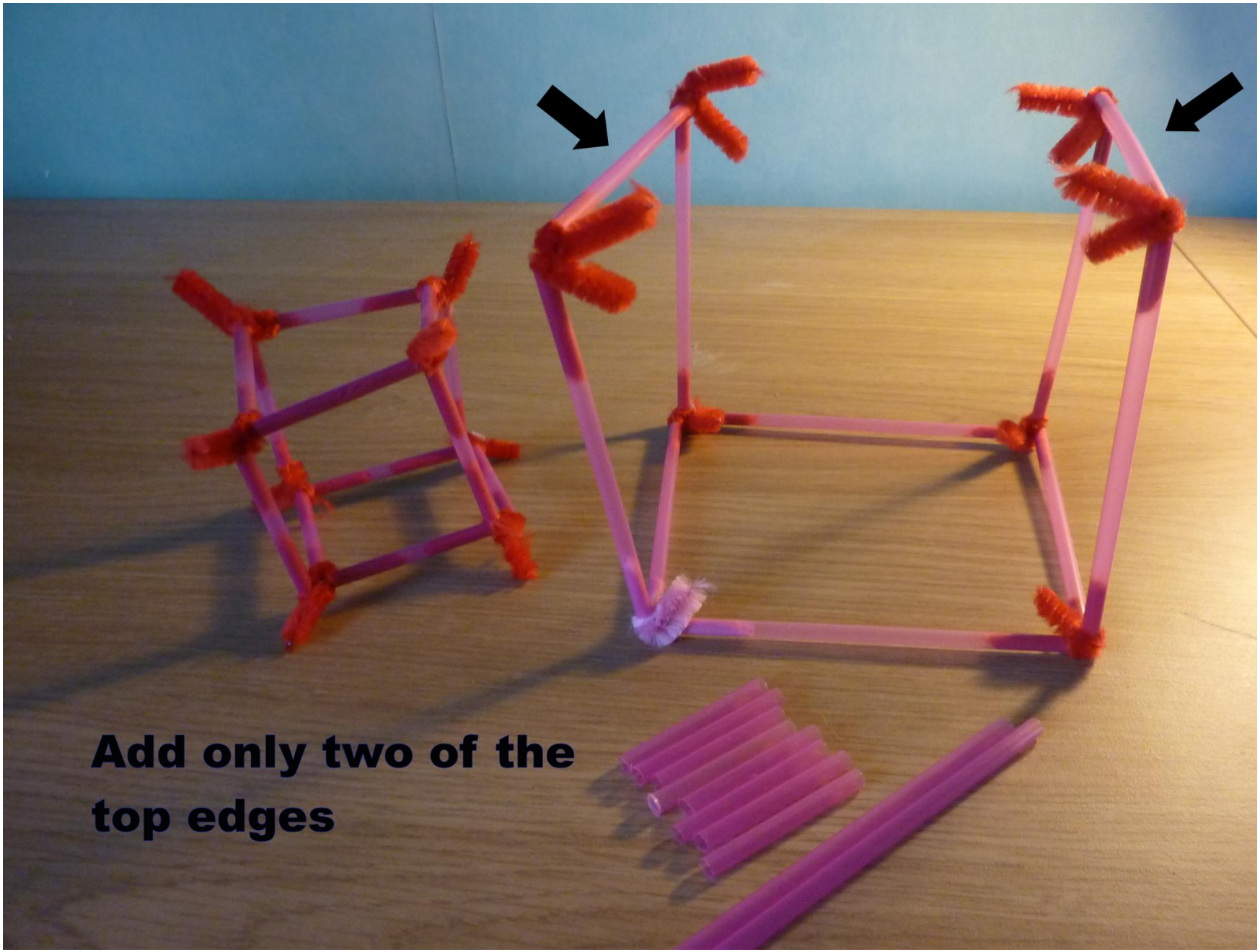

Make a top for the cube
same as the bottom

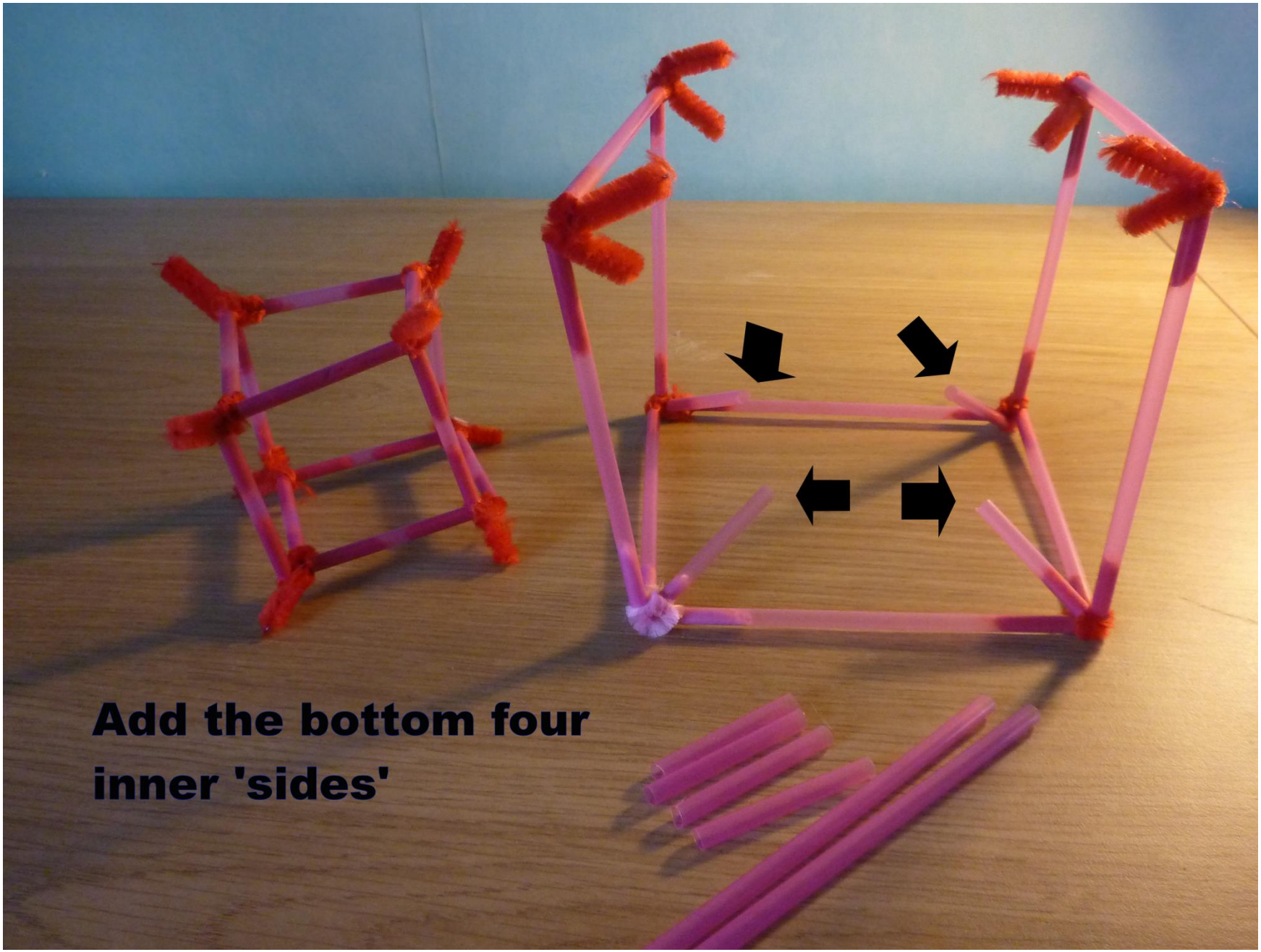

You should now
have the small
inner cube

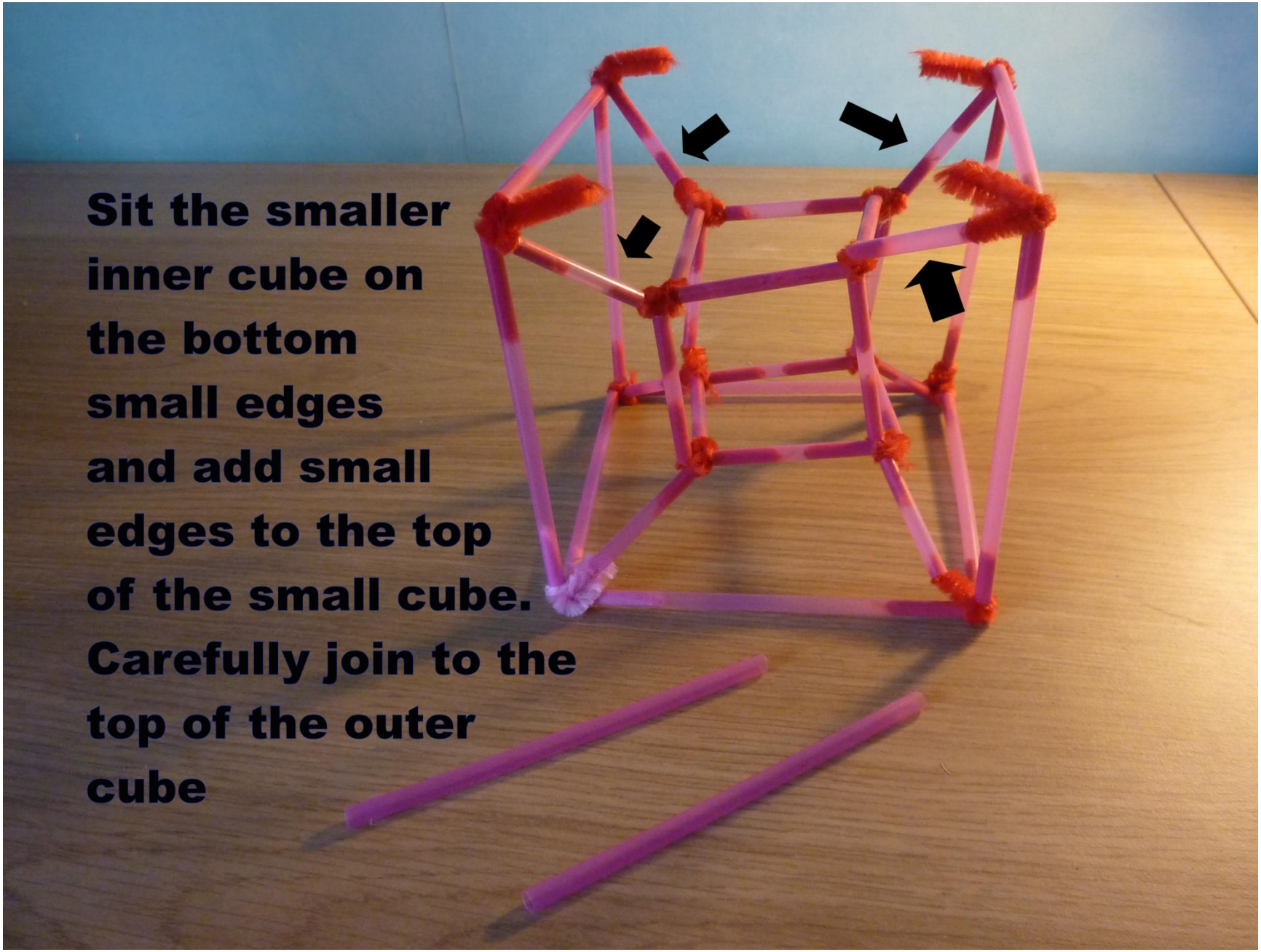

Start to make another outer cube in the same way.
This time the extra pipe-cleaner points inwards

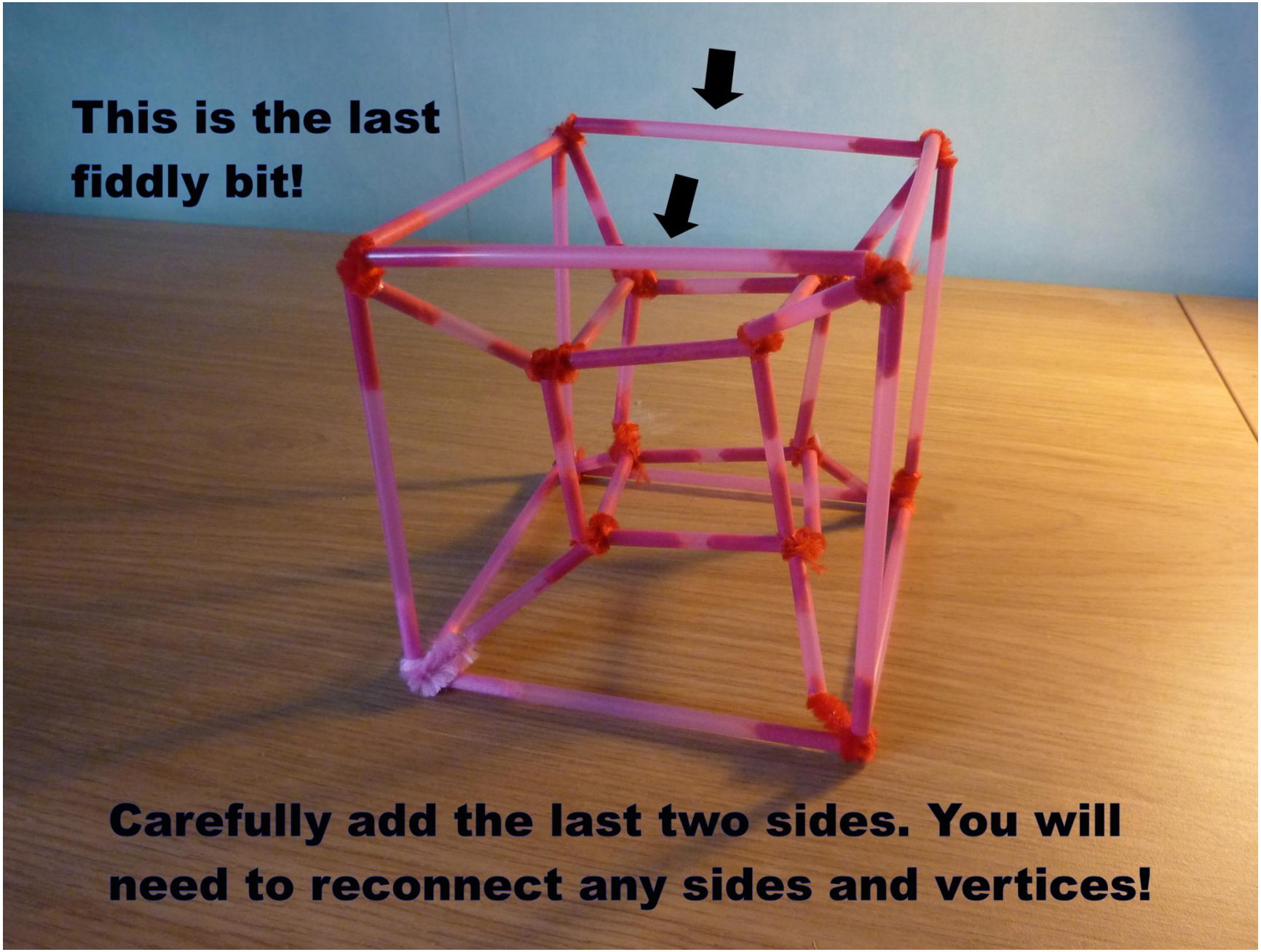

Add the
uprights


Add the top
pipecleaner
vertices

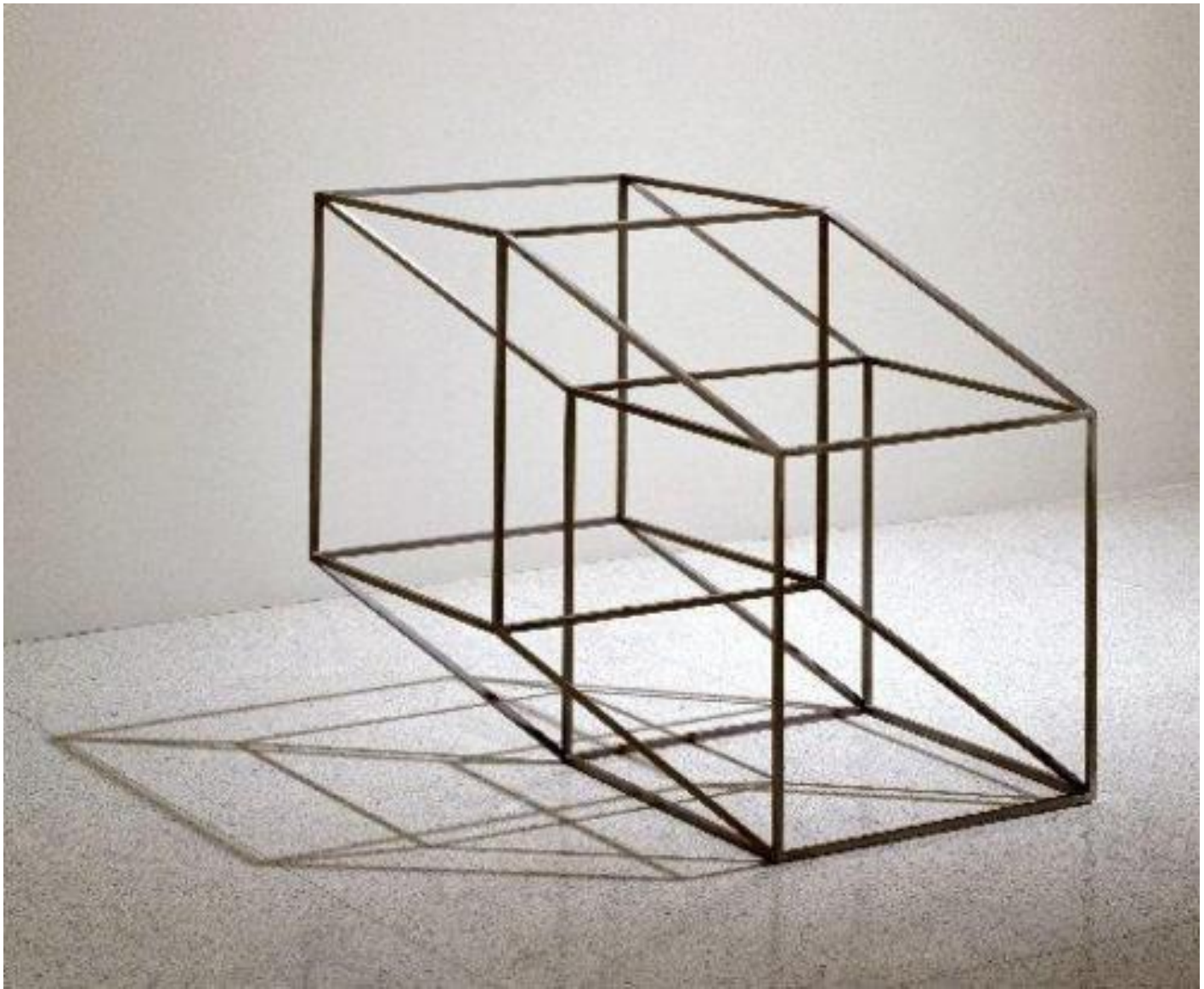

Add only two of the
top edges


Add the bottom four
inner 'sides'


Sit the smaller inner cube on the bottom small edges and add small edges to the top of the small cube. Carefully join to the top of the outer cube


This is the last fiddly bit!
Carefully add the last two sides. You will need to reconnect any sides and vertices!

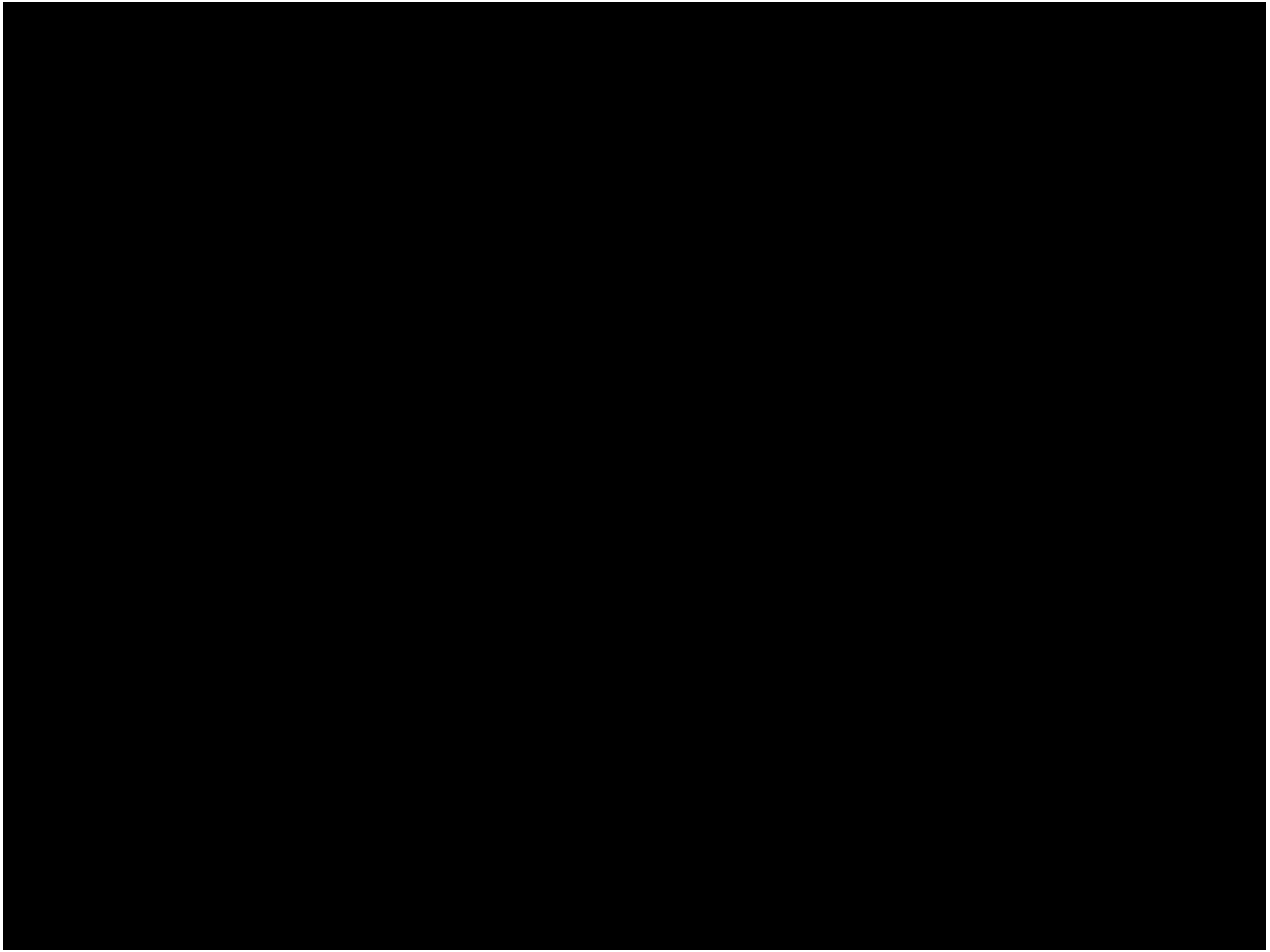
Alternative model – all sides equal



Unfolding the cube into flatland

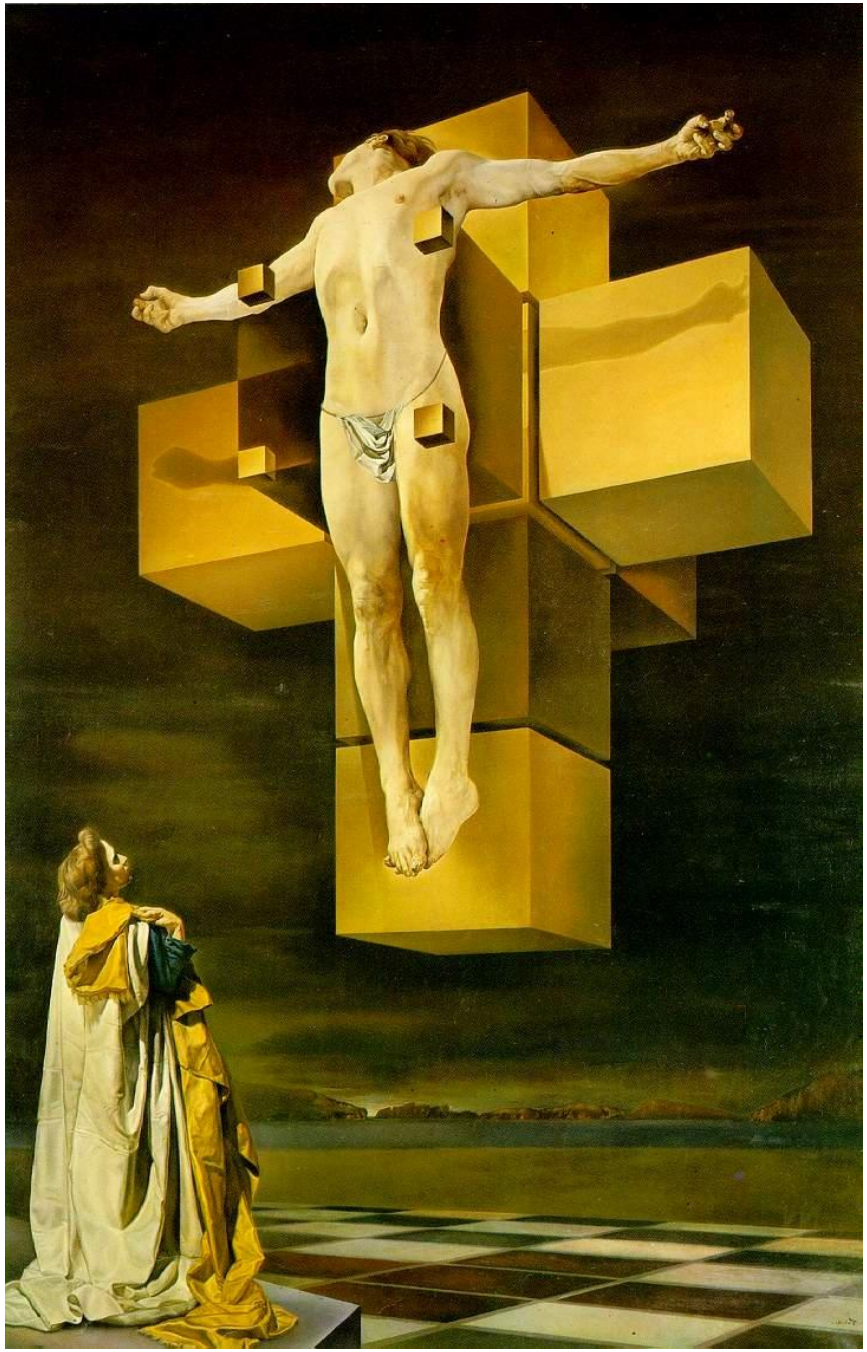
http://www.youtube.com/watch?v=n5Xt2_sTkZQ

Unfolding the Hypercube into our space



http://www.youtube.com/watch?v=BVo2igbFSPE

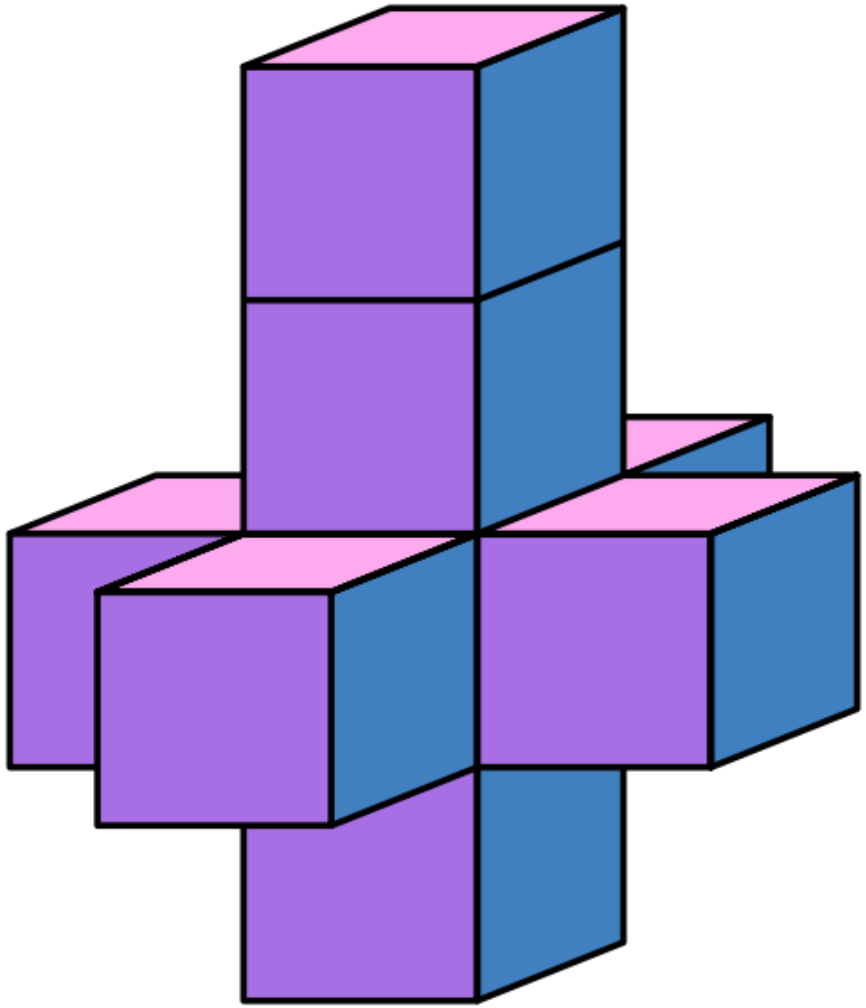
Crucifixion (Corpus Hypercubus) is a 1954 oil-on-canvas painting by Salvador Dalí



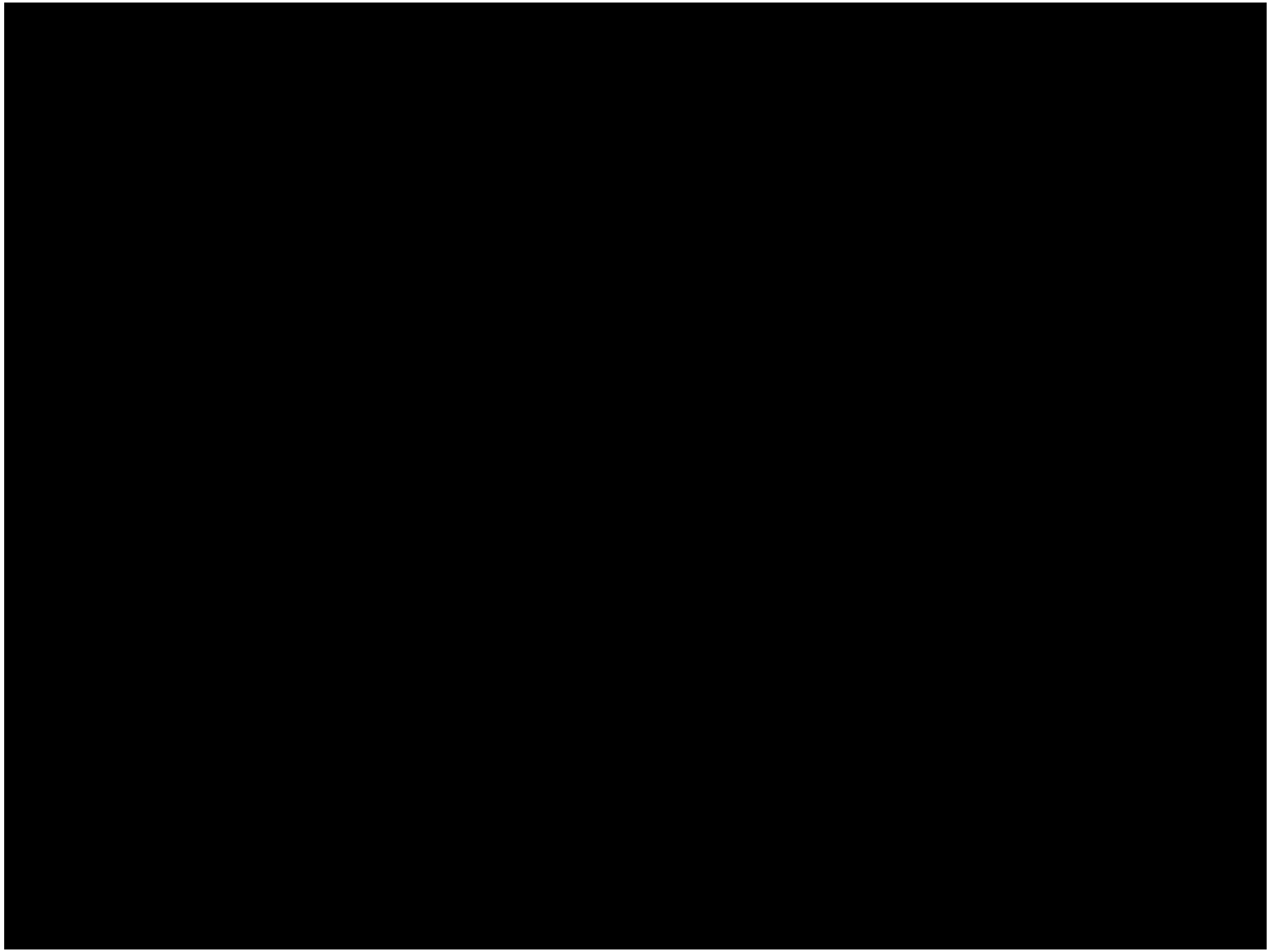
The Hypercube House

"'**—And He Built a Crooked House—**'" is a sci-fi short story by Robert A. Heinlein first published in *Astounding Science Fiction* in February 1941. It was reprinted in the anthology *Fantasia Mathematica* (Clifton Fadiman, ed.) in 1958 and in the Heinlein collection *The Unpleasant Profession of Jonathan Hoag* in 1959. The story is about a mathematically inclined architect named Quintus Teal who has what he thinks is a brilliant idea to save on real estate costs by building a house shaped like the unfolded net of a tesseract. The title is a paraphrase of the nursery rhyme "There Was a Crooked Man".

The house is quickly constructed, in its peculiar "inverted double cross" shape (having eight cubical rooms, arranged as a stack of four cubes with a further four cubes surrounding the second cube up on the stack). However, the night before Teal is to show Bailey and his wife around the house, an earthquake occurs. The three of them arrive the next morning to find what appears to be just a single cubical room. Believing the top seven rooms to have been stolen during the night, they go inside to look for clues.....



Into the abyss? Beyond 4 dimensions



http://www.youtube.com/watch?v=Q_B5GpsbSQw

String theory? An example of 11 dimensions?

Brian Greene:

Making sense of string theory

TED2005 · **Filmed** Feb 2005

Physicist Brian Greene explains superstring theory, the idea that minscule strands of energy vibrating in 11 dimensions create every particle and force in the universe.

http://www.ted.com/talks/brian_greene_on_string_theory

An 8-dimensional model of the universe

- **Garrett Lisi:**
- **An 8-dimensional model of the universe**
- **TED2008** · **Filmed** Feb 2008

Physicist and surfer Garrett Lisi presents a controversial new model of the universe that — just maybe — answers all the big questions. If nothing else, it's the most beautiful 8-dimensional model of elementary particles and forces you've ever seen.

- http://www.ted.com/talks/garrett_lisi_on_his_theory_of_everything

"As of 2012, there is still no hard evidence that nature is described by a Grand Unified Theory."

Last entry on Wikipedia, October 13th, 2014